Worksheet #5 – PHY102 (Spring 2011)

Solving Equations and Differential Equations II

Last week you learned the following functions, which are among the most important in Mathematica:

`Solve` – solves equations analytically
`NSolve` – solves polynomial equations numerically
`FindRoot` – solves equations numerically (Requires a starting point)
`DSolve` – solves differential equations analytically
`NDSolve` – solves differential equations numerically

Once you know what these routines do and how to use them, you have a very powerful set of tools for solving problems in physics. However the hardest part of physics is to set up the mathematical description of the problem, and that you still need to do by hand. This worksheet is intended to help you learn some more about setting up and solving physics problems.

Problem 1.

First review the various method to solve equations:
(i) Use `Plot[{Exp[Sqrt[x]], 2*x}, {x, ??, ??}]` with appropriate choices of the limits to roughly estimate the values of x for which `Exp[Sqrt[x]] = 2 x`.
(ii) Use `NSolve` to find numerical values of the solutions.
(iii) Use `Solve` followed by `N` to find the same numerical values.
(iv) Use `FindRoot` to again find the numerical values of the solutions.

Problem 2.

A ball is falling vertically through a fluid. In addition to gravity (use $g = 9.80\,\mathrm{m/s^2}$), a drag force F_d acts on the ball. The drag force opposes the motion and increases in proportion to the velocity: $\vec{F_d} = -k\vec{v}$, where k is a *drag coefficient* that depends on the fluid. (This may or may not be an accurate physics approximation—that depends on the how big the velocity is, and for typical applications involving air resistance, a force proportional to v^2 would be much closer to the truth. But it is a very *convenient* approximation, because it simplifies the mathematics by decoupling the equations of motion in the x and y directions.)

(i) Find and plot the time dependence of the position and velocity of a 100 g ball that is released from rest at $t = 0$ in a fluid with drag coefficient $k = 0.02$. Choose a time range that shows the approach to terminal velocity of the ball. Hint: it is easiest to solve this problem by solving for the motion analytically using DSolve.

(ii) Find the terminal velocity to an accuracy of 1 part in 10^5.

Problem 3.

Consider a cannon at the top of a 500 m high hill. Assume that the cannon fires 0.1 kg cannonballs horizontally with initial velocity $500 m/s$, and $k = 0.01$.

(i) How long does it take the cannonball to reach the ground?

(ii) Find the range (= horizontal distance traveled).

(iii) If the cannon is fired at an angle θ above the horizontal, what angle gives the maximum range, and what is that range?