

Worksheet #12 – PHY102 (Spring 2011)

Review Exercises — Part 1 of 2

These are examples of questions with the degree of difficulty that you can expect to encounter in your lab exam during the last week of class. You must use Mathematica to solve all parts of the problems.

I. Vectors

For arbitrary vectors \vec{a} , \vec{b} , \vec{c} ,

1. Show that $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$.
2. Show that $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$.
3. Show that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

II. Matrices

Enter the following matrix and display it in MatrixForm.

$$\mathbf{m} = \begin{pmatrix} 0.2 & -0.4 & 0.1 \\ 0.4 & 0.2 & -0.3 \\ 0.6 & -0.3 & -0.1 \end{pmatrix}$$

1. Find the Transpose of \mathbf{m} .
2. Find the Determinant of \mathbf{m} .
3. Find the Trace of \mathbf{m} .
4. Find the Eigenvalues and Eigenvectors of \mathbf{m} .

You will encounter solving eigenvalue problems in mechanics, when you study small oscillations of systems with more than one degree of freedom; and also in quantum mechanics. In those applications, the matrix is always symmetric, and as a result the eigenvalues are always real. This example doesn't have that simplicity, but Mathematica should be able to handle it OK — just don't be surprised if complex numbers appear in the answer!

5. Show that the sum of the eigenvalues of this matrix is equal to the Trace of \mathbf{m} .
6. Show that the product of the eigenvalues is equal to the Determinant of \mathbf{m} .

III. Function definitions

Define $\mathbf{f} = \mathbf{0.8x} + \mathbf{x^2} - \mathbf{3.2x^4}$.

1. Find df/dx and d^2f/dx^2 .
2. Find $\int f dx$.
3. Find $\int_{-1}^{\sqrt{2}} f(x) dx$.
4. Find the four roots of the equation $f = 0$ and check that each of these roots actually satisfies the equation.
5. Define $\mathbf{f[x_]} := \mathbf{0.8x} + \mathbf{x^2} - \mathbf{3.2x^4}$, and repeat the previous four exercises using the function $\mathbf{f(y)}$.