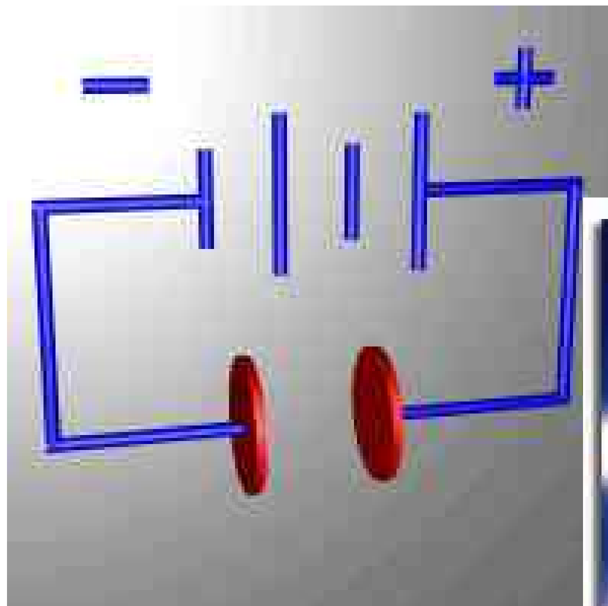
Electrical energy & Capacitance



PHY232 – Spring 2008

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(original ppt courtesy of Remco Zegers)

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work…

- A force is conservative if the work done on an object when moving from A to B does not depend on the path followed. Consequently, work was defined as:

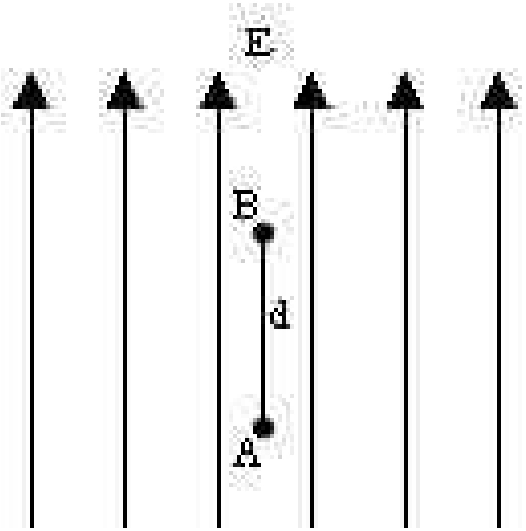
$$W = PE_i - PE_f = -\Delta PE$$

- This was derived in Phy231 for a gravitational force, but as we saw in the previous chapter, gravitational and Coulomb forces are very similar:

  $F_g=Gm_1m_2/r_{12}^2$ with $G=6.67\times10^{-11}$ Nm^2/kg^2
  $F_e=k_eq_1q_2/r_{12}^2$ with $k_e=8.99\times10^{9}$ Nm^2/C^2

Hence: The Coulomb force is a conservative force

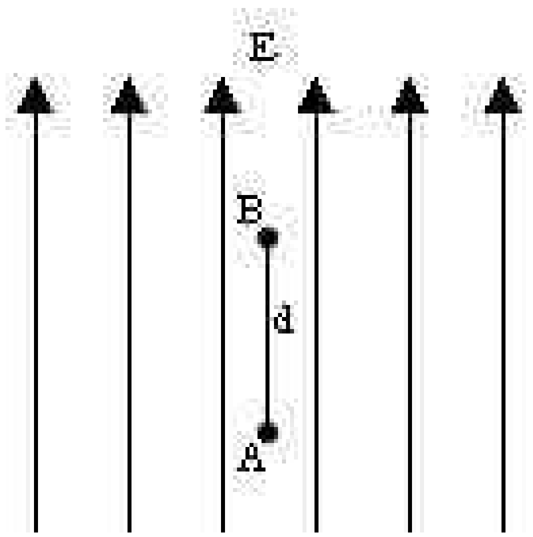
work & potential energy



- consider a charge +q moving in an E field from A to B over a distance D. We can ignore gravity (why?)
- What is the work done by the field?
- What is the change in PE?
- If initially at rest, what is its speed at B?

- $W_{AB}=Fd\cos\theta$ with θ the angle between F and direction of movement, so
  - $W_{AB}=Fd$
  - $W_{AB}=qEd$ (since $F=qE$)
  - work done BY the field ON the charge (W is positive)
- $\Delta PE=-W_{AB}=-qEd$: negative, so the potential energy has decreased
- Conservation of energy:
  - $\Delta PE+\Delta KE=0$
  - $\Delta KE=1/2m(v_f^2-v_i^2)$
  - $1/2mv_f^2=qEd$
  - $v=\sqrt{(2qEd/m)}$

work & potential energy II

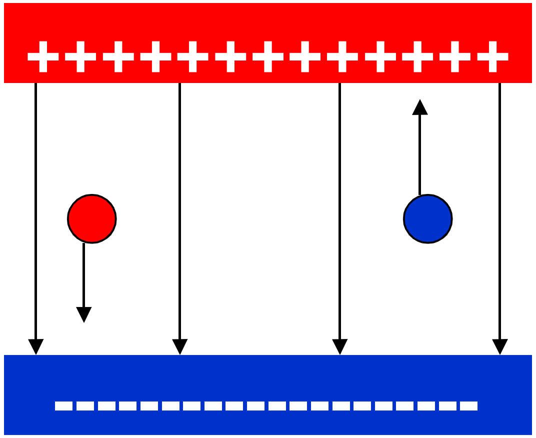


- Consider the same situation for a charge of –q.
- Can it move from A to B without an external force being applied, assuming the charge is initially (A) and finally (B) at rest?

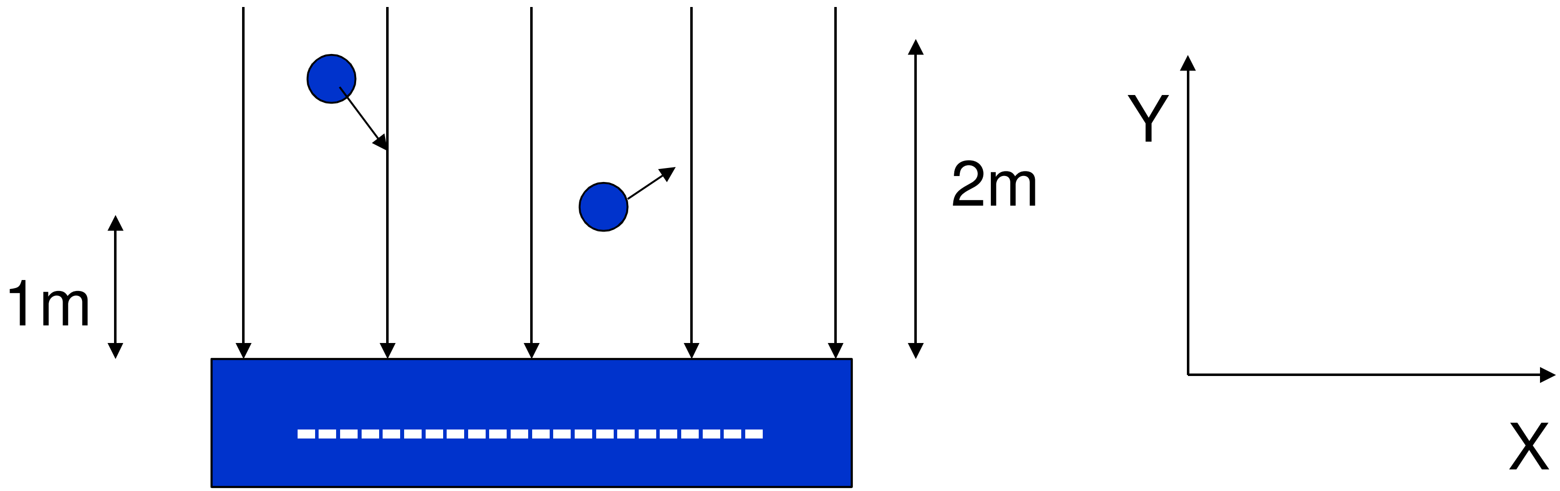
- $W_{AB}=-qEd$; negative, so work must be done by the charge. This can only happen if an external force is applied
- Note: if the charge had an initial velocity the energy could come from the kinetic energy (I.e. it would slow down)
- If the charge is at rest at A and B: external work done: -qEd
- If the charge has final velocity v then external work done: $W=1/2mv_f^2+|q|Ed$

Conclusion

- In the absence of external forces, a positive charge placed in an electric field will move along the field lines (from + to -) to reduce the potential energy
- In the absence of external forces, a negative charge placed in an electric field will move along the field lines (from - to +) to reduce the potential energy



question



- a negatively charged (-1 μC) mass of 1 g is shot diagonally in an electric field created by a negatively charge plate (E=100 N/C). It starts at 2 m distance from the plate and stops 1 m from the plate, before turning back. What was the initial velocity in the direction along the field lines?

answer

- **Note:** the direction along the surface of the plate does not play a role (there is no force in that direction!)

Kinetic energy balance

- Initial kinetic energy: $1/2mv^2=0.5*0.001*(v_x^2+v_y^2)$
- Final (at turning point) kinetic energy: $0.5*0.001*v_x^2$
- Change in kinetic energy: $\Delta KE=-0.5*0.001*v_y^2=-5x10^{-4}v_y^2$

Potential energy balance

- Change in Potential energy: $\Delta PE=-qEd=1x10^{-6}*100*1=+10^{-4}$ J

- Conservation of energy: $\Delta PE+\Delta KE=0$ so $-5x10^{-4}v_y^2+10^{-4}=0$
- $v_y=0.44$ m/s

Electrical potential

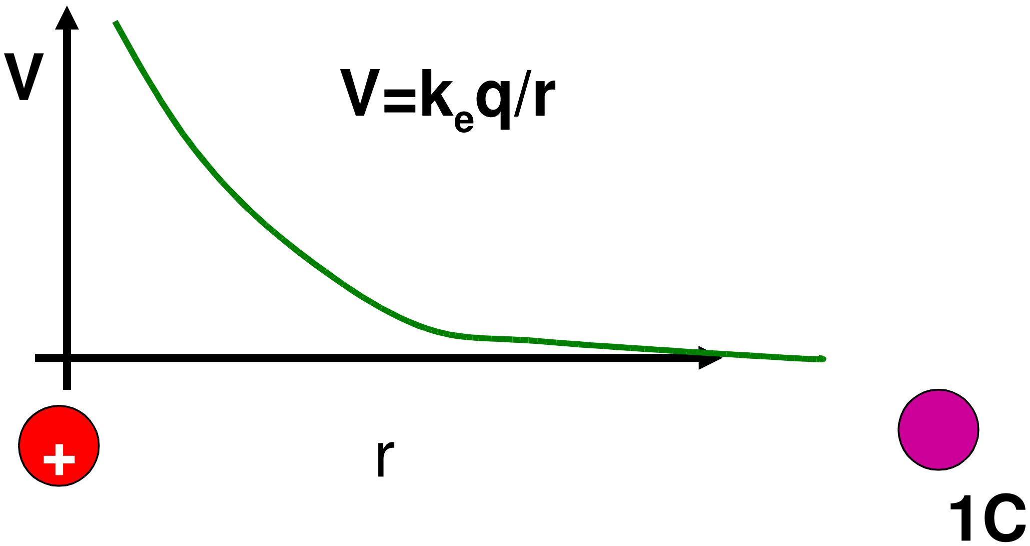
- The change in electrical potential energy of a particle of charge Q in a field with strength E over a distance d depends on the charge of the particle: $\Delta PE=-QEd$

- For convenience, it is useful to define the difference in electrical potential between two points (ΔV), that is independent of the charge that is moving: $\Delta V= \Delta PE/Q=-|E|d$

- The electrical potential difference has units [J/C] which is usually referred to as Volt ([V]). It is a scalar

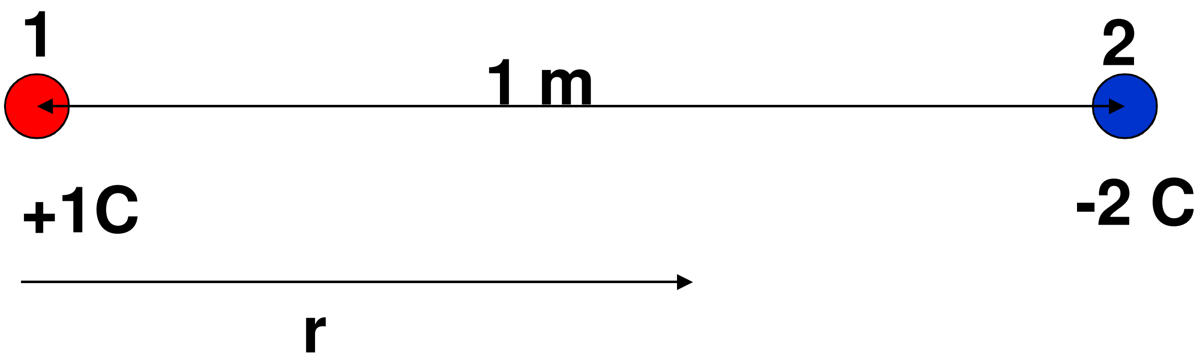
- Since $\Delta V= -Ed$, so $E= -\Delta V/d$ the units of E ([N/C] before) can also be given as [V/m]. They are equivalent, but [V/m] is more often used.

Electric potential due to a single charge



- the potential at a distance r away from a charge +q is the work done in bringing a charge of 1 C from infinity (V=0) to the point r: $V=k_eq/r$
- If the charge that is creating the potential is negative (-q) then $V=-k_eq/r$
- If the field is created by more than one charge, then the superposition principle can be used to calculate the potential at any point

example



a) what is the electric field at a distance r?
b) what is the electric potential at a distance r?

a) $E=E_1 - E_2=k_e(Q_1/r^2)-k_e(Q_2/[1-r]^2)=k_e(1/r^2)-k_e(-2/[1-r]^2)= k_e (1/r^2+2/[1-r]^2)$ Note the -: E is a vector
b) $V=V_1+V_2=k_e(Q_1/r)+k_e(Q_2/[1-r])=k_e(1/r-2/[1-r])$ Note the +: V is a scalar

question

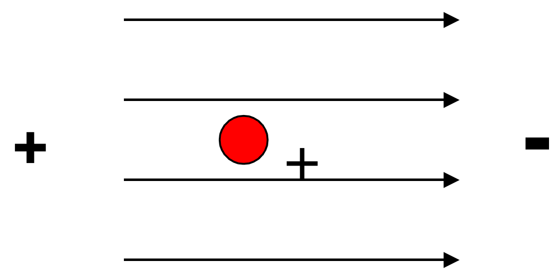
➢ a proton is moving in the direction of the electric field. During this process, the potential energy …… and its electric potential ……

b) increases, decreases

c) decreases, increases

d) increases, increases

e) decreases, decreases

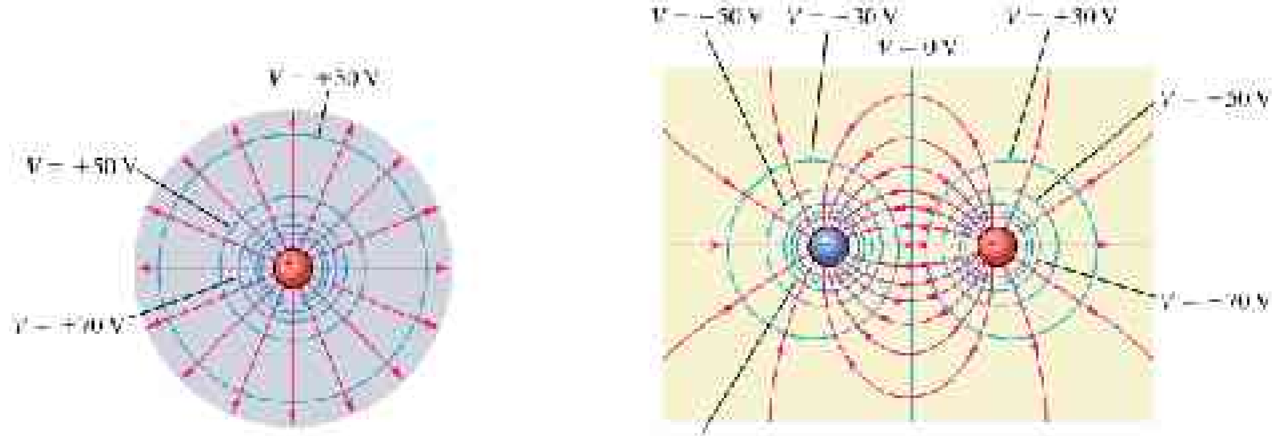


$\Delta PE = -W_{AB} = -qEd$, so the potential energy decreases (proton is positive)

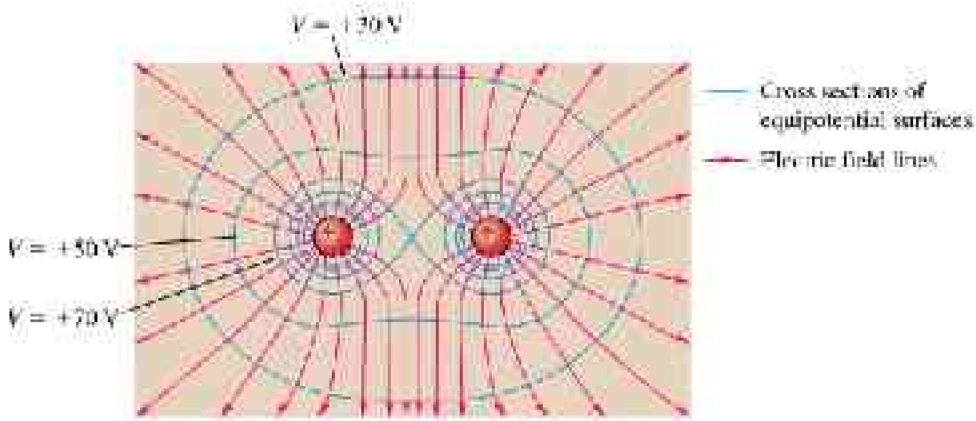
$\Delta V = \Delta PE/q$, so the electric potential that the proton feels decreases

Note: if the proton were exchanged for an electron moving in the same direction, the potential energy would increase (electron is negative), but the electric potential would still decrease since the latter is independent of the particle that is moving in the field

equipotential surfaces



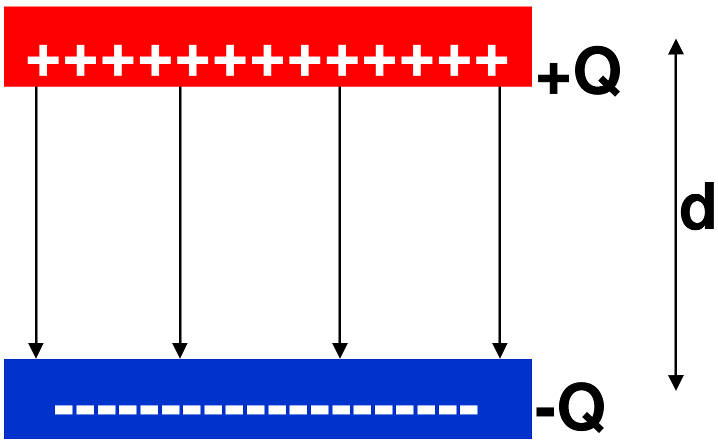
(a) A single positive charge (b) An electric dipole

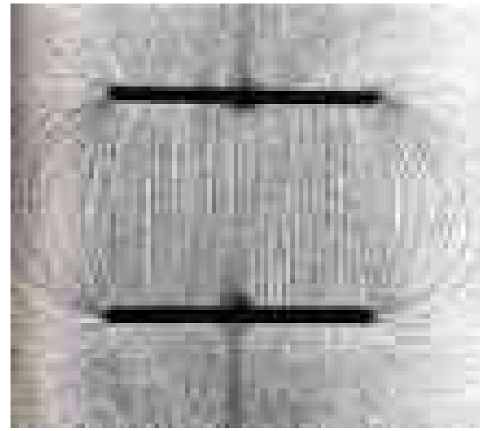


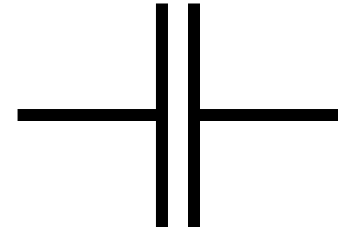
(c) Two equal positive charges

compare with a map

A capacitor



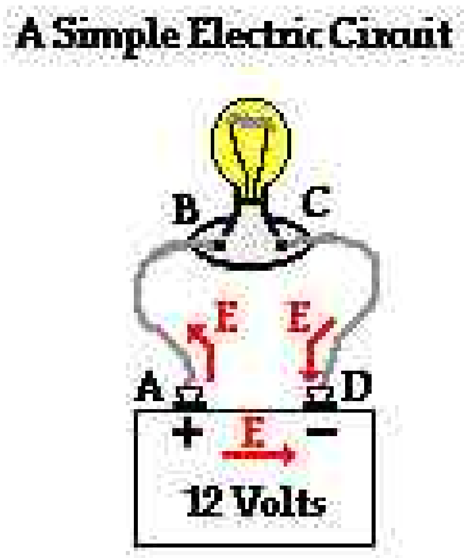




symbol for capacitor when used in electric circuit

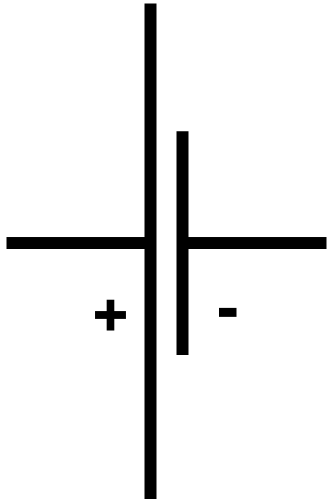
- is a device to create a constant electric field. The potential difference $V=Ed$
- is a device to store charge (+ and -) in electrical circuits.
- the charge stored Q is proportional to the potential difference V: $Q=CV$
- C is the capacitance, units C/V or Farad (F)
- very often C is given in terms of μF (10^{-6}F), nF (10^{-9}F), or pF (10^{-12}F)
- Other shapes exist, but for a parallel plate capacitor: $C=\varepsilon_0 A/d$ where $\varepsilon_0=1/(4\ \mathrm{pi}\ k) = 8.85\times10^{-12}$ F/m and A the area of the plates

electric circuits: batteries

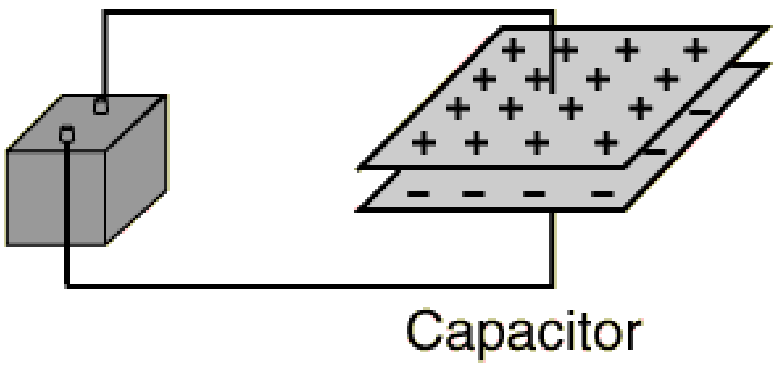


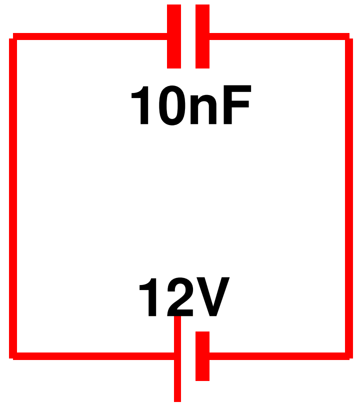
- The battery does work (e.g. using chemical energy) to move positive charge from the – terminal to the + terminal. Chemical energy is transformed into electrical potential energy.
- Once at the + terminal, the charge can move through an external circuit to do work transforming electrical potential energy into other forms

Symbol used in electric circuits:



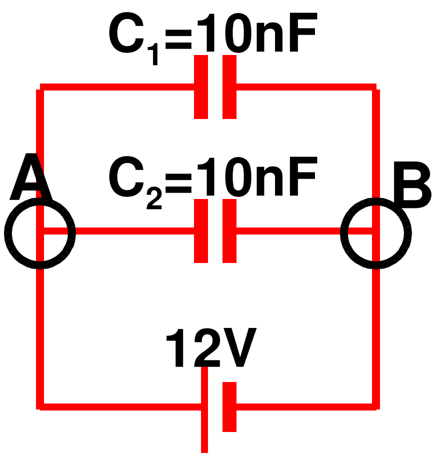
Our first circuit





- The battery will transport charge from one plate to the other until the voltage produced by the charge build-up is equal to the battery charge
- example: a 12V battery is connected to a capacitor of 10 nF. How much charge is stored?
- answer $Q=CV=10\times10^{-9}$ x 12V=120 nC
- NOTE, Q on one plate, -Q on the other (total is 0, but Q is called "the charge")!
- if the battery is replaced by a 300 V battery, and the capacitor is 2000μF, how much charge is stored?
- answer $Q=CV=2000\times10^{-6}$ x 300V=0.6C
  - We will see later that this corresponds to $0.5CV^2$=90 J of energy, which is the same as a 1 kg ball moving at a velocity of 13.4 m/s

capacitors in parallel



At the points ○ the potential is fixed to one value, say 12V at A and 0 V at B

This means that the capacitances C_1 and C_2 must have the same Voltage.

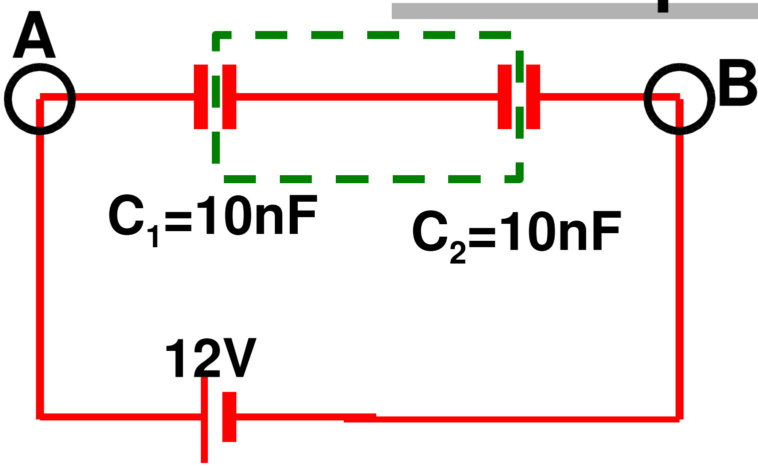
The total charge stored is $Q=Q_1+Q_2$.

- We can replace C_1 and C_2 with one equivalent capacitor:

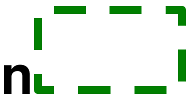
  $Q_1=C_1V$ & $Q_2=C_2V$ is replaced by: $Q=C_{eq}V$

  since $Q=Q_1+Q_2$, $C_1V+C_2V=C_{eq}V$ so:
- $C_{eq}=C_1+C_2$
- This holds for any combination of parallel placed capacitances $C_{eq}=C_1+C_2+C_3+\ldots$
- The equivalent capacitance is larger than each of the components

capacitors in series



The voltage drop of 12V is over both capacitors. $V=V_1+V_2$

The two plates enclosed in  are not connected to the battery and must be neutral on average. Therefore the charge stored in C_1 and C_2 are the same

- we can again replace C_1 and C_2 with one equivalent capacitor but now we start from:

$V=V_1+V_2$ so, $V=Q/C_1+Q/C_2=Q/C_{eq}$ and thus: $1/C_{eq}=1/C_1 + 1/C_2$

- This holds for any combination of in series placed capacitances $1/C_{eq}=1/C_1+1/C_2+1/C_3+\ldots$
- The equivalent capacitor is smaller than each of the components

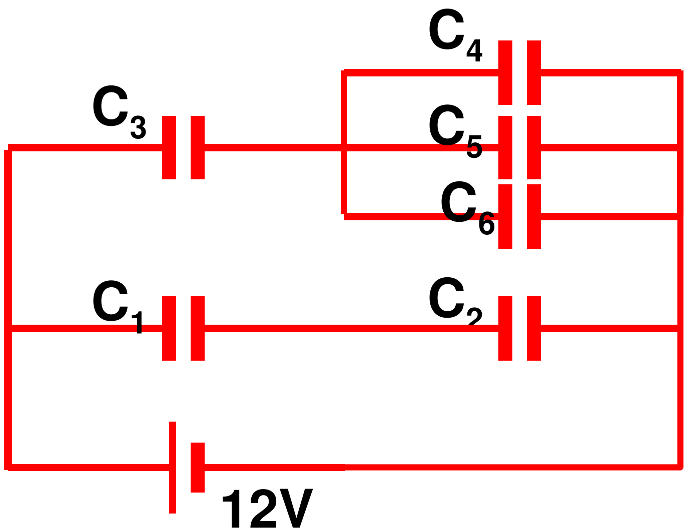
question

- Given three capacitors of 1 nF, an capacitor can be constructed that has minimally a capacitance of:

c) 1/3 nF
d) 1 nF
e) 1.5 nF
f) 3 nF

The smallest possible is by putting the three in series:
$1/C_{eq}=1/C_1+1/C_2+1/C_3=1+1+1=3$ so $C_{eq}=1/3$ nF

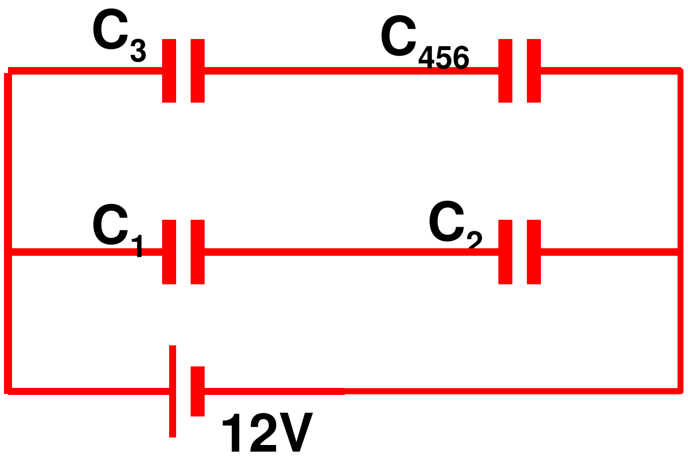
Fun with capacitors: what is the equivalent C?



STRATEGY: replace subgroups of capacitors, starting at the smallest level and slowly building up.

- step 1: C_4 and C_5 and C_6 are in parallel. They can be replaced by once equivalent $C_{456}=C_4+C_5+C_6$

step II



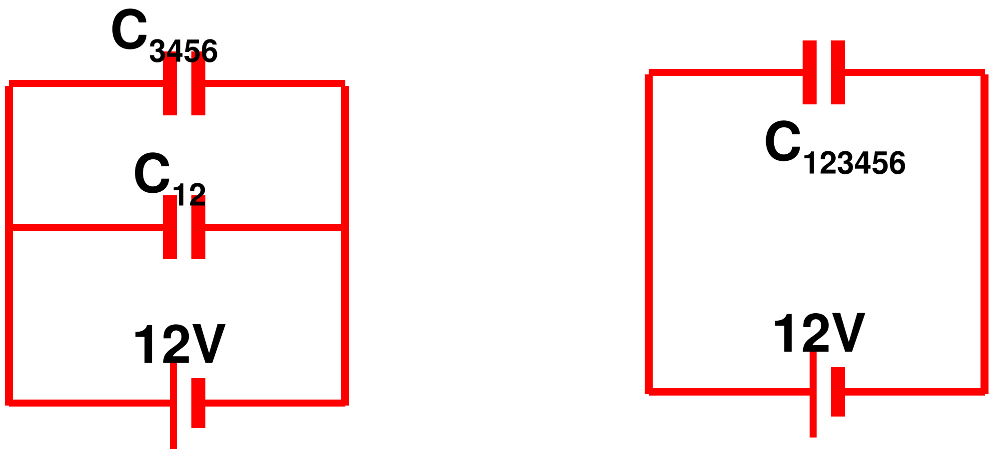
- C_3 and C_{456} are in series. Replace with equivalent C:

  $1/C_{3456}=1/C_3+1/C_{456}$ so $C_{3456}=C_3C_{456}/(C_3+C_{456})$

- C_1 and C_2 are in series. Replace with equivalent C:

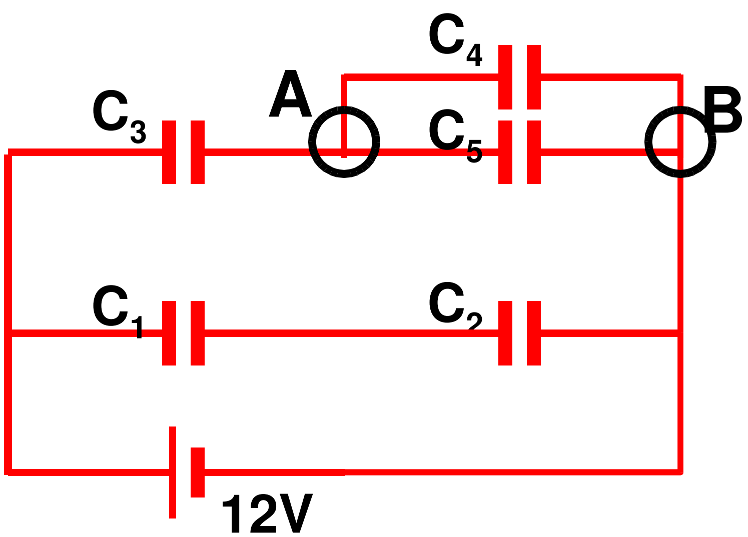
  $1/C_{12}=1/C_1+1/C_2$ so $C_{12}=C_1C_2/(C_1+C_2)$

step III



- C_{12} and C_{3456} are in parallel, replace by equivalent C of $C_{123456}=C_{12}+C_{3456}$

problem

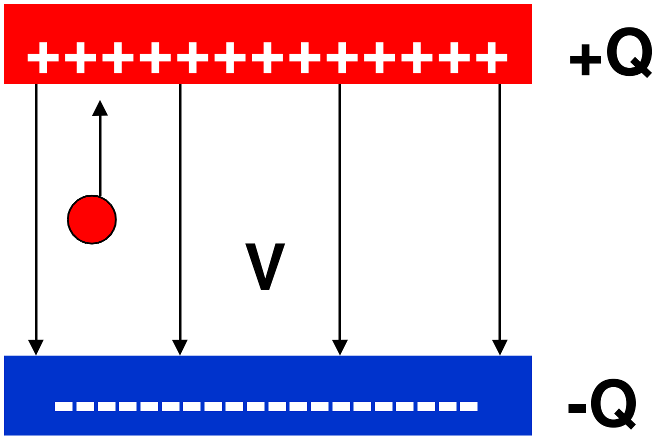


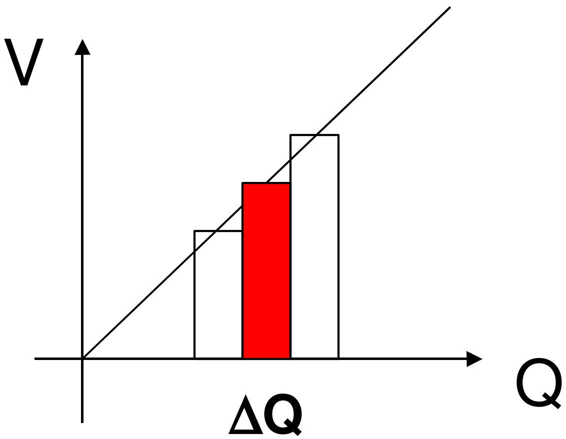
C_1=10nF
C_2=20nF
C_3=10nF
C_4=10nF
C_5=20nF

What is V_{ab}?

- $V_{12}=V_{345}=12V$
- $C_{45}=C_4+C_5=10nF+20nF=30nF$
- $C_{345}=C_3C_{45}/(C_3+C_{45})=300/40=7.5nF$
- $Q_{345}=V_{345}C_{345}=12V*7.5nF=90nC$
- $Q_{45}=Q_{345}$
- $V_{45}=Q_{45}/C_{45}=90nC/30nF=3V$
- check $V_3=Q_3/C_3=Q_{345}/C_3=90nC/10nF=9V$ $V_3+V_{45}=12V$ okay!

energy stored in a capacitor





- the work done transferring a small amount ΔQ from – to + takes an amount of work equal to $\Delta W = V\Delta Q$
- At the same time, V is increased, since $V=(Q+\Delta Q/C)$
- The total work done when moving charge Q starting at V=0 equals: $W=1/2QV=1/2(CV)V=1/2CV^2$
- **Therefore, the amount of energy stored in a capacitor equals:**

$$E_C=1/2\ C\ V^2$$

example

- A parallel-plate capacitor is constructed with plate area of 0.40 m^2 and a plate separation of 0.1mm. How much energy is stored when it is charged to a potential difference of 12V?

answer:

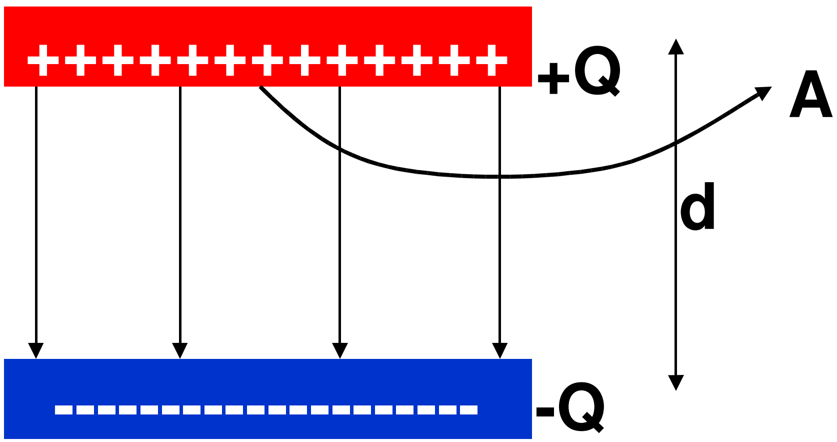
First calculate $C=\varepsilon_0 A/d=8.85\times10^{-12}$ x 0.40 / 0.0001=3.54×10^{-8} F

Energy stored: $E=1/2CV^2=0.5\times3.54\times10^{-8}\times12^2=2.55\times10^{-6}$ J

Now let's assume a 2000μF capacitor being charged with a 300V battery: $E=1/2CV^2=90$J

This is similar to a ball of 1 kg being fired at 13.4 m/s!!

capacitors II



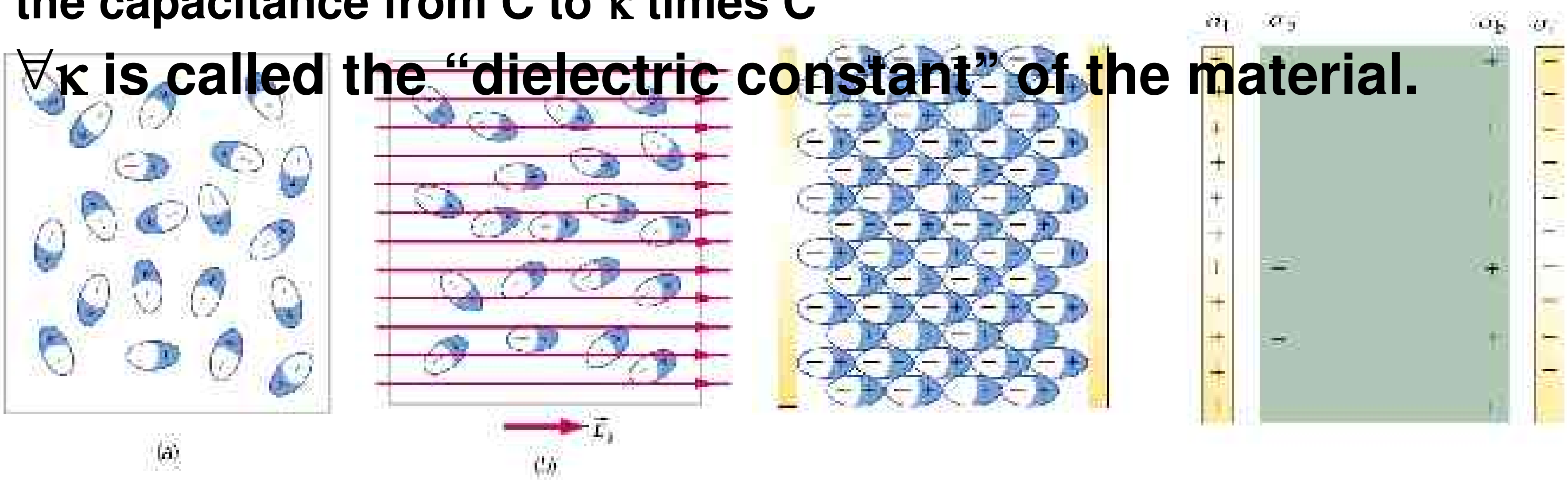
material	κ
vacuum	1.00000
air	1.00059
glass	5.6
paper	3.7
water	80

- the charge density of one of the plates is defined as: $\sigma=Q/A$
- The equation $C=\varepsilon_0 A/d$ assumes the area between the plates is in vacuum (free space)
- If the space is replaced by an insulating material, the constant ε_0 must be replaced by $\kappa\varepsilon_0$ where κ (kappa) is the dielectric constant for that material, relative to vacuum
- Therefore: $C=\kappa\varepsilon_0 A/d$

Inserting a "Dielectric"

• **molecules, such as those in glass, can be polarized**
• **when placed in an E-field, they orient themselves along the field lines; the negative plates attract the positive side of the molecules**
•**near to positive plate, net negative charge is collected; near the negative plate, net positive charge is collected.**
•**If no battery is connected, the initial potential difference V between the plates will drop to V/κ.**
•**If a battery was connected, more charge can be added, increasing the capacitance from C to κ times C**

∀κ is called the "dielectric constant" of the material.



problem

- **An amount of 10 J is stored in a parallel plate capacitor with C=10nF. Then the plates are disconnected from the battery and a plate of material is inserted between the plates. A voltage drop of 1000 V is recorded. What is the dielectric constant of the material?**

answer:

step 1: $E_c=1/2CV^2$ so $10=0.5\times 10\times10^{-9}\,V^2$, V=44721 V

step 2: after disconnecting and inserting the plate, the voltage over the capacitor is equal to $V_{original}/\kappa$

So: $(44721-1000)=44721/\kappa$

$\kappa=1.023$

problem

- An ideal parallel plate capacitor is connected to a battery and becomes fully charged. The capacitor is then disconnected and the separation between the plates is increased in such a way that no charge leaks off. The energy stored in the capacitor has

b) increased

c) decreased

d) not changed

e) become zero

answer: $E_c=1/2CV^2$ with $C=\kappa\varepsilon_0 A/d$. If d increases, C becomes smaller. The charge remains the same and Q=CV. So, if C becomes smaller, V becomes larger by the same factor.
Rewrite: $E_c=1/2CV^2=1/2QV$. Since Q is constant and V goes up, E_c must increase.

Remember

- **Electric force (Vector!) acting on object 1 (or 2): $F=k_e q_1 q_2/r_{12}^2$**

- **Electric field (Vector!) due to object 1 at a distance r: $E=k_e q_1/r^2$**

- **Electric potential (Scalar!) at a distance r away from a charge q_1:**

  $V=k_e q_1/r$