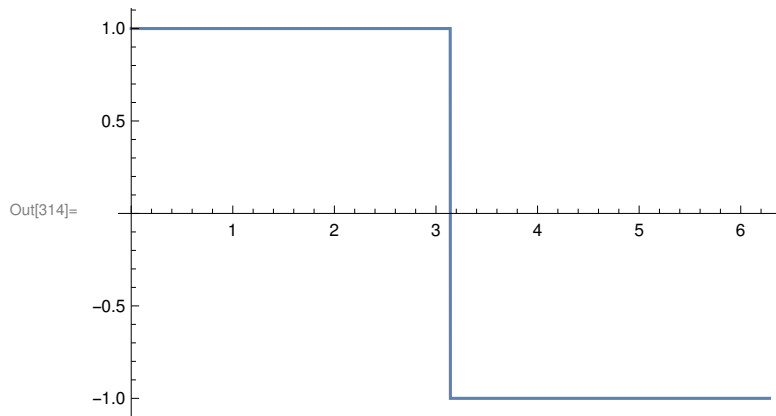


# FourierSeriesExamples

```
In[312]:= Clear["Global`*"]
```

```
In[313]:= F = 1 - 2 * UnitStep[t - Pi/omega]  
Plot[F /. omega -> 1, {t, 0, 2 * Pi}]
```

```
Out[313]= 1 - 2 UnitStep[- $\frac{\pi}{\omega}$  + t]
```



```
In[315]:= An = (omega / (2 * Pi)) *  
Integrate[F * Exp[-I * n * omega * t], {t, 0, 2 * Pi / omega}, Assumptions -> {omega > 0}]  
Ann = Simplify[An, Assumptions -> {n ∈ Integers}]
```

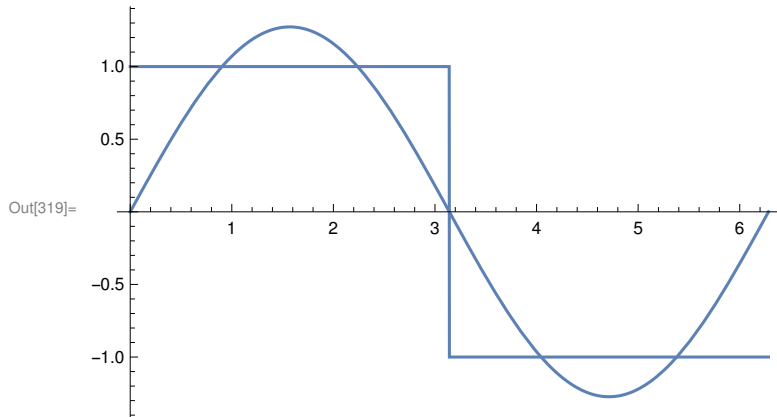
```
Out[315]= - $\frac{i e^{-2 i n \pi} (-1 + e^{i n \pi})^2}{2 n \pi}$ 
```

```
Out[316]=  $\frac{i (-1 + (-1)^n)}{n \pi}$ 
```

```
In[317]:= F1 = Sum[Ann * Exp[I * omega * n * t], {n, 1, 1}] +
          Sum[Ann * Exp[I * omega * n * t], {n, -1, -1}]
FF1 = ExpToTrig[F1]
Plot[{FF1, F} /. omega -> 1, {t, 0, 2 * Pi}]
```

Out[317]=  $\frac{2 i e^{-i \omega t}}{\pi} - \frac{2 i e^{i \omega t}}{\pi}$

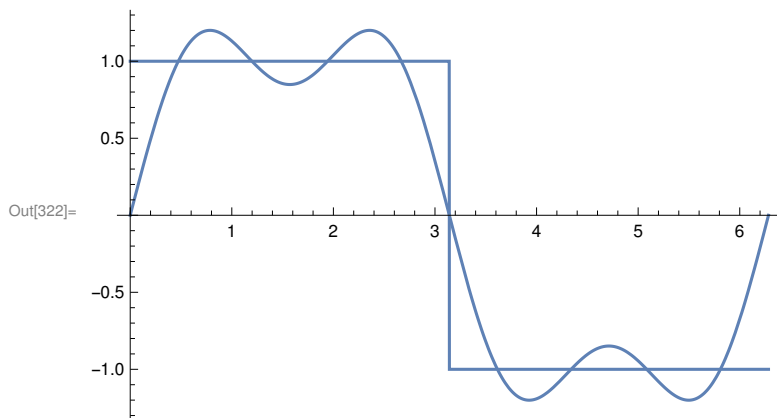
Out[318]=  $\frac{4 \text{Sin}[\omega t]}{\pi}$



```
In[320]:= F3 = Sum[Ann * Exp[I * omega * n * t], {n, 1, 3}] +
          Sum[Ann * Exp[I * omega * n * t], {n, -3, -1}]
FF3 = ExpToTrig[F3]
Plot[{FF3, F} /. omega -> 1, {t, 0, 2 * Pi}]
```

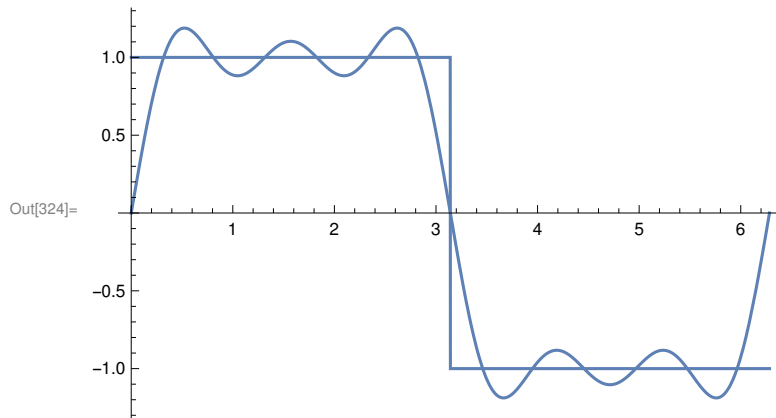
Out[320]=  $\frac{2 i e^{-i \omega t}}{\pi} - \frac{2 i e^{i \omega t}}{\pi} + \frac{2 i e^{-3 i \omega t}}{3 \pi} - \frac{2 i e^{3 i \omega t}}{3 \pi}$

Out[321]=  $\frac{4 \text{Sin}[\omega t]}{\pi} + \frac{4 \text{Sin}[3 \omega t]}{3 \pi}$



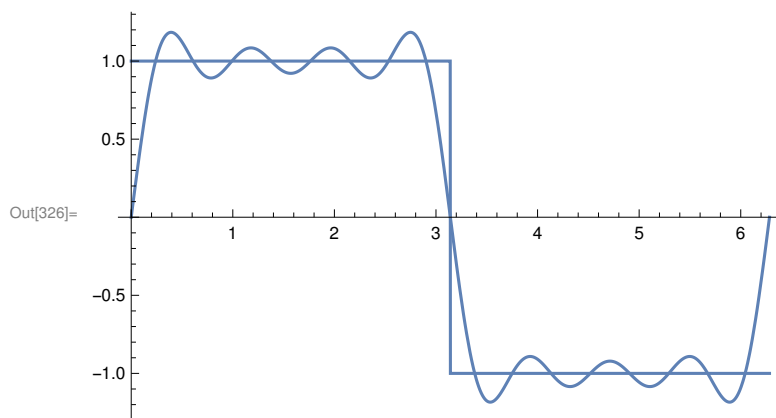
```
In[323]:= FF5 = ExpToTrig[
  Sum[Ann * Exp[I * omega * n * t], {n, 1, 5}] + Sum[Ann * Exp[I * omega * n * t], {n, -5, -1}]
  Plot[{FF5, F} /. omega -> 1, {t, 0, 2 * Pi}]
```

$$\text{Out[323]= } \frac{4 \sin[\omega t]}{\pi} + \frac{4 \sin[3 \omega t]}{3 \pi} + \frac{4 \sin[5 \omega t]}{5 \pi}$$



```
In[325]:= FF7 = ExpToTrig[
  Sum[Ann * Exp[I * omega * n * t], {n, 1, 7}] + Sum[Ann * Exp[I * omega * n * t], {n, -7, -1}]
  Plot[{FF7, F} /. omega -> 1, {t, 0, 2 * Pi}]
```

$$\text{Out[325]= } \frac{4 \sin[\omega t]}{\pi} + \frac{4 \sin[3 \omega t]}{3 \pi} + \frac{4 \sin[5 \omega t]}{5 \pi} + \frac{4 \sin[7 \omega t]}{7 \pi}$$

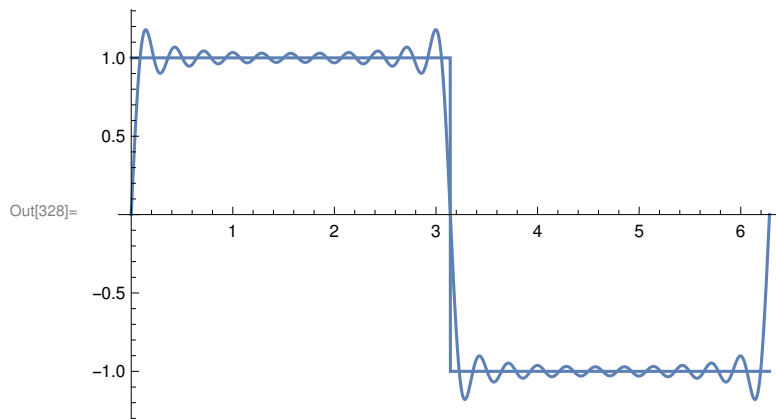


```
In[327]:= FF21 = ExpToTrig[Sum[Ann * Exp[I * omega * n * t], {n, 1, 21}] +
      Sum[Ann * Exp[I * omega * n * t], {n, -21, -1}]]
Plot[{FF21, F} /. omega -> 1, {t, 0, 2 * Pi}]
```

$$\text{Out[327]= } \frac{4 \sin[\omega t]}{\pi} + \frac{4 \sin[3 \omega t]}{3 \pi} + \frac{4 \sin[5 \omega t]}{5 \pi} +$$

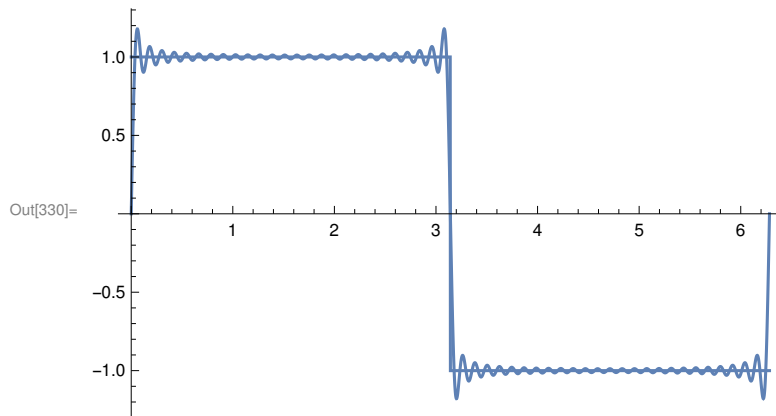
$$\frac{4 \sin[7 \omega t]}{7 \pi} + \frac{4 \sin[9 \omega t]}{9 \pi} + \frac{4 \sin[11 \omega t]}{11 \pi} + \frac{4 \sin[13 \omega t]}{13 \pi} +$$

$$\frac{4 \sin[15 \omega t]}{15 \pi} + \frac{4 \sin[17 \omega t]}{17 \pi} + \frac{4 \sin[19 \omega t]}{19 \pi} + \frac{4 \sin[21 \omega t]}{21 \pi}$$



```
In[329]:= FF51 = ExpToTrig[Sum[Ann * Exp[I * omega * n * t], {n, 1, 51}] +
Sum[Ann * Exp[I * omega * n * t], {n, -51, -1}]]
Plot[{FF51, F} /. omega -> 1, {t, 0, 2 * Pi}]
```

$$\begin{aligned} \text{Out[329]= } & \frac{4 \sin[\omega t]}{\pi} + \frac{4 \sin[3 \omega t]}{3 \pi} + \frac{4 \sin[5 \omega t]}{5 \pi} + \\ & \frac{4 \sin[7 \omega t]}{7 \pi} + \frac{4 \sin[9 \omega t]}{9 \pi} + \frac{4 \sin[11 \omega t]}{11 \pi} + \frac{4 \sin[13 \omega t]}{13 \pi} + \\ & \frac{4 \sin[15 \omega t]}{15 \pi} + \frac{4 \sin[17 \omega t]}{17 \pi} + \frac{4 \sin[19 \omega t]}{19 \pi} + \\ & \frac{4 \sin[21 \omega t]}{21 \pi} + \frac{4 \sin[23 \omega t]}{23 \pi} + \frac{4 \sin[25 \omega t]}{25 \pi} + \frac{4 \sin[27 \omega t]}{27 \pi} + \\ & \frac{4 \sin[29 \omega t]}{29 \pi} + \frac{4 \sin[31 \omega t]}{31 \pi} + \frac{4 \sin[33 \omega t]}{33 \pi} + \frac{4 \sin[35 \omega t]}{35 \pi} + \\ & \frac{4 \sin[37 \omega t]}{37 \pi} + \frac{4 \sin[39 \omega t]}{39 \pi} + \frac{4 \sin[41 \omega t]}{41 \pi} + \frac{4 \sin[43 \omega t]}{43 \pi} + \\ & \frac{4 \sin[45 \omega t]}{45 \pi} + \frac{4 \sin[47 \omega t]}{47 \pi} + \frac{4 \sin[49 \omega t]}{49 \pi} + \frac{4 \sin[51 \omega t]}{51 \pi} \end{aligned}$$

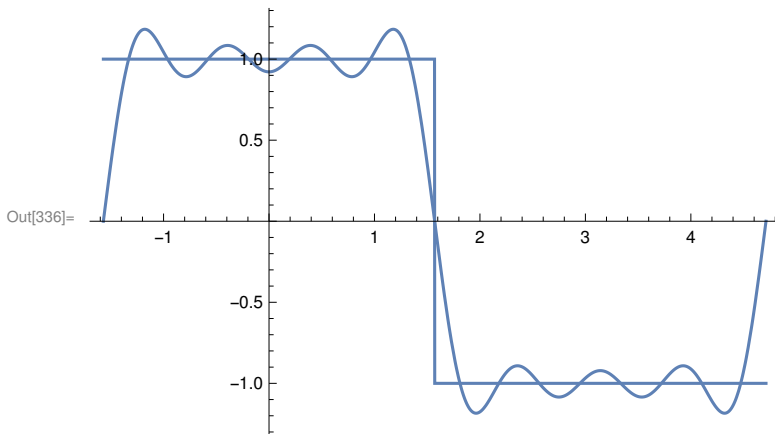


```
In[333]:= G = F /. t -> tt + Pi / (2 * omega)
GG7 = Expand[Simplify[FF7 /. t -> tt + Pi / (2 * omega)]]
```

$$\text{Out[333]= } 1 - 2 \text{UnitStep}\left[-\frac{\pi}{2 \omega} + tt\right]$$

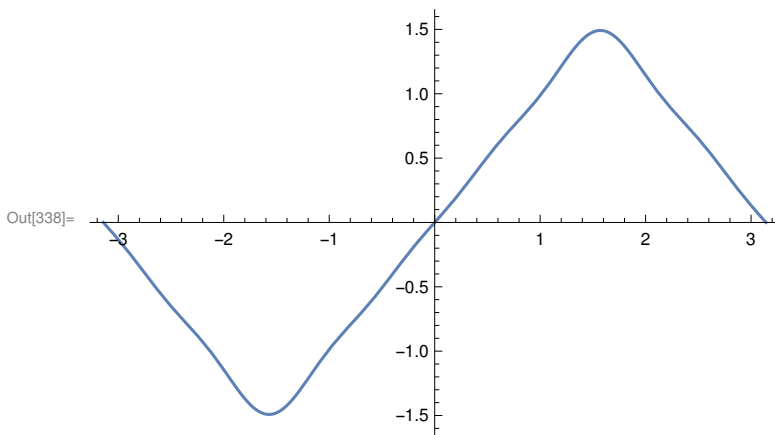
$$\text{Out[334]= } \frac{4 \cos[\omega tt]}{\pi} - \frac{4 \cos[3 \omega tt]}{3 \pi} + \frac{4 \cos[5 \omega tt]}{5 \pi} - \frac{4 \cos[7 \omega tt]}{7 \pi}$$

```
In[336]:= Plot[{GG7, G} /. omega -> 1, {tt, -Pi/2, 3 * Pi/2}]
```



```
In[337]:= HH7 = Expand[Integrate[GG7, tt]]
Plot[HH7 /. omega -> 1, {tt, -Pi, Pi}]
```

Out[337]= 
$$\frac{4 \sin[\omega tt]}{\omega \pi} - \frac{4 \sin[3 \omega tt]}{9 \omega \pi} + \frac{4 \sin[5 \omega tt]}{25 \omega \pi} - \frac{4 \sin[7 \omega tt]}{49 \omega \pi}$$



```
In[341]:= Expand[HH7 / (4 / (omega * Pi))]
```

Out[341]= 
$$\sin[\omega tt] - \frac{1}{9} \sin[3 \omega tt] + \frac{1}{25} \sin[5 \omega tt] - \frac{1}{49} \sin[7 \omega tt]$$