Physics 321 – Spring 2017

Homework #3, Due at beginning of class Wednesday Feb 1.

1. [6 pts] A simple pendulum consists of a point mass M hanging from a massless string of length R and swinging in a vertical plane. Its maximum angle is 90°, i.e., it was released from rest from a point where the string was horizontal. Let θ be its angle with respect to the vertical, so $\theta = 0$ corresonds to the lowest point of its arc.

   (a) Find the equation of motion that relates $\ddot{\theta}$ to $\sin\theta$ by writing "F = ma" in the tangential ($\hat{\theta}$) direction.

   (b) Integrate the equation of motion numerically using Mathematica, including the initial conditions $\theta = \pi/2$ and $\dot{\theta} = 0$ at $t = 0$, to find the time it takes for the pendulum to travel from $\theta = 90°$ to $\theta = 45°$.

   (c) Integrate the equation of motion numerically using Mathematica, including the initial conditions $\theta = \pi/2$ and $\dot{\theta} = 0$ at $t = 0$, to find the time it takes for the pendulum to travel from $\theta = 45°$ to $\theta = 0°$. Perhaps you will want to do this by finding the time it takes to travel from $\pi/2$ to 0 and then subtracting the time calculated in part (b).

   (Note that you calculated the same two times in HW02, using a method based on energy conservation.)

2. [6 pts] A particle of mass M is moving in a plane, with its Cartesian coordinates (x, y) given by

$$
\begin{aligned}
x &= A\,[Bt - \sin(Bt)] \\
y &= A\,[1 - \cos(Bt)]
\end{aligned}
$$

   where A and B are positive constants.

   (a) Find the times at which the speed is a maximum.

   (b) Find the tangential component of acceleration, i.e., the component of acceleration in the direction of motion, as a function of the time t.

   (c) Find the "radial" component of acceleration, i.e., the magnitude of the component of acceleration that is perpendicular to the direction of motion. (You can do this by first finding a unit vector that is perpendicular to the velocity direction; or you can calculate it from the magnitude of the acceleration vector and its tangential component.)

3. [8 pts] A particle with electric charge Q and mass M is traveling in a region where there is a constant electric field of magnitude E and a constant magnetic field of magnitude B. Both the electric and the magnetic field point in the z direction. Assume the initial conditions at $t = 0$ are given by $x = y = z = 0$, $v_x = v_x^0$, $v_y = 0$, and $v_z = v_z^0$.

   (a) Write the x, y, and z components of the equation of motion.

   (b) Solve the equations of motion to find the velocity as a function of time.

   (c) Find the position of the particle as a function of time.

(Last updated 1/26/2017.)