Physics 321 – Spring 2017

Homework #8, Due at beginning of class Wednesday March 15.

1. [4 pts] Taylor problem 5.2

2. [4 pts] Taylor problem 5.28

3. [4 pts] A hook is at height y above the floor, where y is constant for all negative times: $y = y_0$ for $t < 0$. For positive times, y oscillates: $y = y_0 + A \sin \omega t$ for $t > 0$. A mass M hangs from an ideal massless spring attached to this hook. The mass is at height x above the floor. The mass hangs motionless at $x = x_0 = y_0 - Mg/k$ for $t < 0$, where k is the spring constant. Let $\omega_0 = \sqrt{k/M}$ as usual. You can figure out what the unstretched length of the spring is from the information given.

   (a) Find the motion $x(t)$ of the mass for $t > 0$ if $\omega = 2\omega_0$.

   (b) Find the motion $x(t)$ of the mass for $t > 0$ if $\omega = \omega_0$.

   The efficient and elegant way to do this problem is to first solve it for an arbitrary driving frequency ω, and then set $\omega = 2\omega_0$ for part (a) and take the limit $\omega \to \omega_0$ for part (b). But if you're chicken, you could just treat (a) and (b) as two separate problems.

4. [4 pts] A driven harmonic oscillator obeys the equation $\ddot{x} + x = t\,(A - t)$ for $0 < t < A$. Given the initial conditions $x = \dot{x} = 0$ at $t = 0$, find the subsequent motion $x(t)$ during the time interval $0 < t < A$.

5. [4 pts] A damped driven harmonic oscillator obeys the equation $\ddot{x} + 2\beta\dot{x} + x = t\,e^{-\alpha t}$ for $t > 0$, where $0 < \beta < 1$ and α is a positive constant.

   Given the initial conditions $x = \dot{x} = 0$ at $t = 0$, find the subsequent motion $x(t)$. (Hint: as is so often the case, the easiest way to solve the differential equation is to guess the answer.)

(Last updated 3/1/2017.)