Physics 321 – Spring 2017

Homework #9, Due at beginning of class Friday March 24

1. [20 pts] A "triangle wave" can be defined by $F(t) = 1 - 2\omega|t|/\pi$ for $-\pi/\omega < t < +\pi/\omega$, with $F(t)$ defined at all other values of the time t by the property of having period $2\pi/\omega$.

   (a) Find the Fourier series representation of this $F(t)$. Express your answer BOTH in exponential form and in the form of sines and/or cosines. But get the exponential form first (it's easier); and then get the sines+cosines version from that.

   (b) Solve the driven damped oscillator equation $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F(t)$ in the form of an infinite series. You will find the exponential form of your answer to part (a) most convenient, so your answer here will be a sum from $n = -\infty$ to ∞ of terms proportional to $e^{i\,n\,\omega\,t}$. You can neglect the solutions to the homogeneous equation, which contains two arbitrary constants, because those "transient" effects go away like $e^{-\beta t}$ if you wait long enough.

   (c) Convert your answer for part (b) to a sum of $\sin(n\,\omega\,t)$ and $\cos(n\,\omega\,t)$ terms with coefficients that are obviously real.

   (d) Explicitly write out the $\sin(n\,\omega\,t)$ and $\cos(n\,\omega\,t)$ terms in part (c) for $n = 1$, $n = 2$, and $n = 3$ for the case $\omega = \omega_0/3$. Which term is most important if β is small?

   Math reminders: You can separate real and imaginary parts using

$$\frac{1}{a + bi} = \frac{1}{a + bi} \times \frac{a - bi}{a - bi} = \frac{a - bi}{a^2 + b^2}$$

$$e^{i\,a} = \cos(a) + i\,\sin(a)$$

(Last updated 3/30/2016.)