Phy 321 Spring 2017

HW 10.1

1. $F(t) = t$ for $-\frac{\pi}{\omega} < t < \frac{\pi}{\omega}$

$$F(t) = \sum_{n=-\infty}^{\infty} A_n e^{in\omega t}$$

$$\int_{-\frac{\pi}{\omega}}^{\pi/\omega} F(t)\, e^{-im\omega t}\, dt = \sum_{n=-\infty}^{\infty} A_n \int_{-\pi/\omega}^{\pi/\omega} e^{i(n-m)\omega t}\, dt$$

$$= A_m \left(\frac{2\pi}{\omega}\right)$$

$$A_n = \frac{\omega}{2\pi} \int_{-\frac{\pi}{\omega}}^{\pi/\omega} t\, e^{-in\omega t}\, dt$$

$(A_0 = 0)$

$u = t \qquad dv = e^{-in\omega t}$

$du = dt \qquad v = \frac{e^{-in\omega t}}{-in\omega}$

$$A_n = \frac{\omega}{2\pi} \left\{ \left[ \frac{t\, e^{-in\omega t}}{-in\omega} \right]_{-\pi/\omega}^{\pi/\omega} - \int_{-\pi/\omega}^{\pi/\omega} dt\, \frac{e^{-in\omega t}}{-in\omega} \right\}$$

HW / 10.2

$$A_n = \left(\frac{\omega}{2\pi}\right)\left\{\frac{\left(\frac{\pi}{\omega}\right) e^{-in\pi} - \left(\frac{-\pi}{\omega}\right) e^{in\pi}}{-in\omega}\right.$$

$$\left. - \left(\frac{1}{-in\omega}\right)\left[\frac{e^{-in\omega t}}{-in\omega}\right]_{-\frac{\pi}{\omega}}^{\pi/\omega}\right\}$$

$$= \left(\frac{\omega}{2\pi}\right)\left(\frac{2\pi}{\omega}\right)\frac{(-1)^n}{-in\omega}$$

$$\boxed{A_n = \frac{i(-1)^n}{n\omega}}$$

$-\infty < n < \infty$

But $n \neq 0$

$$F(t) = \sum_{\substack{n=-\infty \\ (\text{not } 0)}}^{\infty} A_n e^{+in\omega t}$$

$$= \sum_{n=1}^{\infty} A_n e^{in\omega t} + \underbrace{\sum_{n=-\infty}^{-1} A_n e^{in\omega t}}_{\sum_{n=1}^{\infty} A_n e^{-in\omega t}}$$

HW 10.3

$$F(t) = \sum_{n=1}^{\infty} \frac{i(-1)^n}{n\omega}(\cos n\omega t + i \sin n\omega t)$$

$$+ \sum_{n=1}^{\infty} \frac{i(-1)^n}{(-n\omega)}(\cos n\omega t - i \sin n\omega t)$$

$$= \sum_{n=1}^{\infty} \frac{-(-1)^n 2}{n\omega} \sin(n\omega t)$$

$$\boxed{F(t) = \left(\frac{-2}{\omega}\right) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin(n\omega t)}$$

2. $$\int \underbrace{(y'^2 + y y' + y^2)}_{F} dx$$

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = \left(\frac{\partial F}{\partial y}\right)$$

$$\frac{d}{dx}(2y' + y) = y' + 2y$$

$$2y'' + \cancel{y'} = \cancel{y'} + 2y$$

HW10/0.4

$y'' = y$

$y = c_1 e^{x} + c_2 e^{-x}$

$x = y = 0$ at one End, so $c_1 + c_2 = 0$

so $c_2 = -c_1$

$y = c_1(e^{x} - e^{-x})$

$x = 1, y = 1$ at other End

$1 = c_1(e^{1} - e^{-1})$

$c_1 = \dfrac{1}{e^{1} - e^{-1}}$

$$\boxed{y = \frac{e^{x} - e^{-x}}{e^{1} - e^{-1}}}$$

or $\boxed{y = \dfrac{\sinh(x)}{\sinh(1)}}$