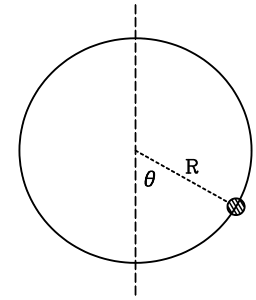
Physics 321 – Spring 2017

Homework #11, Due at beginning of class Wednesday April 5.



1. [5 pts] A bead of mass M slides without friction on a circular loop of wire in a vertical plane, with the force of gravity pointing toward the bottom of the paper. The wire is rotating at constant angular velocity ω about its vertical axis.

   (a) Write the Lagrangian ($L = T - V$) for this problem as a function of the angle θ and $\dot{\theta}$.

   (b) Use your Lagrangian to find the equation of motion for θ. You do not need to solve that equation.

   (c) Is the energy $T + V$ constant for this problem? Explain why or why not.

2. [5 pts] A point mass M slides without friction on a cone of half-angle α:

$$\begin{aligned} x &= \rho \cos(\phi) \\ y &= \rho \sin(\phi) \\ z &= \rho / \tan(\alpha) \end{aligned}$$

   The z-axis points upward, so the potential energy is $M\,g\,z$.

   (a) Write the Lagrangian. It will be a function of ρ, ϕ, $\dot{\rho}$, $\dot{\phi}$; it will not depend on z because z can be related to ρ.

   (b) Write the equations of motion. (There will be two of them: $\frac{d}{dt}\frac{\partial L}{\partial \dot{\rho}} = \frac{\partial L}{\partial \rho}$ and $\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}} = \frac{\partial L}{\partial \phi}$.)

3. [5 pts] Do a modified version of Taylor problem 7.29 in which the massless rod of length ℓ with point mass m at the end is replaced by a uniform rod of length ℓ and mass M. Your goal is to find the equation of motion for the angle of the rod with respect to the vertical direction. You do not need to solve that equation.

4. [5 pts] Taylor problem 7.40, Part (a) and Part (b) only.

(Last updated 3/31/2017.)