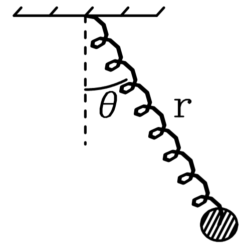
Physics 321 – Spring 2017

Homework #12, Due at beginning of class Wednesday April 12.

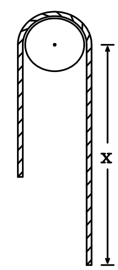
1. [6 pts] A point mass M is attached to the ceiling by a massless spring that has spring constant k and unstretched length B.



(a) Write the Lagrangian for this system using r and θ as the coordinates. Assume the motion lies in the plane of the paper.

(b) Use your Lagrangian to find the second-order differential equations of motion. You do not have to solve those equations.

2. [6 pts] A thin flexible rope of length b and constant mass per unit length σ hangs over a pulley. The pulley has radius R and moment of inertia I.



(a) Write the kinetic energy in terms of $\dot{x}$ and the constants given.

(b) Write the gravitational potential energy in terms of x and the constants given.

(c) Use the Lagrangian to obtain the differential equation of motion for x.

(d) Take the time derivative of the energy conservation equation $E = T + V$, and check that the result is consistent with your result for part (c).

(e) Solve the equation of motion assuming the rope starts at $x = x_0$, with velocity zero, at time $t = 0$.

3. [8 pts] One end of a uniform rod of mass M and length ℓ is constrained to oscillate in the vertical direction: $x_0 = 0$, $y_0 = R\sin(\omega t)$. The center of the rod is at

$$\begin{aligned} x_{\rm cm} &= x_0 + (\ell/2)\sin(\phi) \\ y_{\rm cm} &= y_0 + (\ell/2)\cos(\phi) \end{aligned}$$

(a) Write the Lagrangian $L = T - V$ for this system, where $T = (1/2)M(\dot{x}_{\rm cm}^2 + \dot{y}_{\rm cm}^2) + (1/2)(M\ell^2/12)\dot{\phi}^2$ and $V = M\,g\,y_{\rm cm}$.

(b) Write the equation of motion: $\dot{p} = F$, where $p = \dfrac{\partial L}{\partial \dot{\phi}}$ and $F = \dfrac{\partial L}{\partial \phi}$.

(c) Let $M = 1$, $g = 1$, $\ell = 1$, $R = 0.05$. Try various values of ω, and use Mathematica to find the motion using

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sol = NDSolve[{xxxx == 0, phi[0] == 0.4, phi'[0] == 0}, phi[t], {t, 0, 20}]
Plot[phi[t] /. sol[[1]], {t, 0, 20}]
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where "xxxx" is the equation of motion, to find how the system moves if it starts at $\phi[0] = 0.4$, $\phi'[0] = 0$. For what values of ω does the system oscillate in a stable fashion about the vertical direction $\phi = 0$?

(Last updated 4/10/2017 ($\dot{x}_{\text{cm}}^2 \to \dot{y}_{\text{cm}}^2$ in problem 3).)