Physics 321 – Spring 2017

Homework #13, Due at beginning of class Friday April 21.

Both of these problems involve the motion of a point particle moving under the influence of a central force. Because of angular momentum conservation, the motion lies in a plane. It is convenient to use polar coordinates to describe the motion: the Lagrangian does not depend on the angular coordinate ϕ except through $\dot{\phi}$, which leads to conservation of angular momentum. The total energy is conserved. Combining energy conservation and angular momentum conservation leads to a simple "Effective Potential" equation that relates r to $\dot{r}$. That equation is the key to solving these problems.

1. [10 pts] A particle of mass M moves under the influence of a central force $F = -A/r^3$ where $A > 0$.

   (a) Find the potential $U(r)$ that corresponds to this force.

   (b) Find the "Effective Potential" equation that relates r to $\dot{r}$.

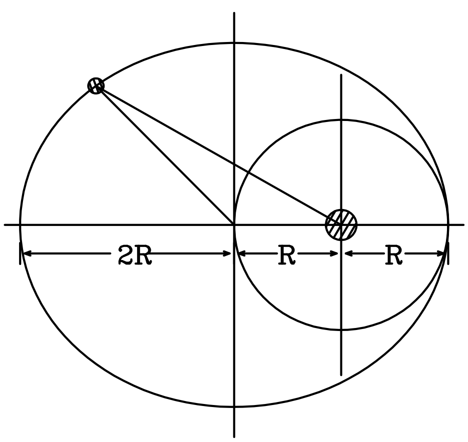
   (c) Find the condition on the angular momentum ℓ that is necessary to have a bounded orbit (i.e., motion that does not go to $r = \infty$).

   (d) Find r as a function of time, assuming $r = r_0$ and $\dot{r} = 0$ at $t = 0$. Instead of using the constant ℓ, express your answer in terms of the initial velocity v_0 where $\ell = M v_0 r_0$. Assume v_0 is small enough that the orbit is bounded. You may find it convenient to introduce the constant

$$b = \sqrt{\frac{A}{M r_0^2} - v_0^2}$$

   (e) Find ϕ as a function of time for the case given in part (d). Assume $\phi = 0$ at $t = 0$.

2. [10 pts] A satellite of small mass m is in a circular orbit of radius R about a planet of large mass M. At a certain time, a powerful rocket motor on the satellite is fired briefly, so as to increase its velocity without changing its direction of motion. As a result, the satellite goes into an elliptical orbit that is tangent to the original circular orbit at the point where the burn took place as shown in the figure. You can neglect the change in mass m caused by the rocket exhaust, and you can assume $m \ll M$ so the planet can be treated as motionless.



   (a) Find the satellite's velocity in the circular orbit in terms of G, M, and R.

   (b) Find the satellite's velocity just after the burn in terms of G, M, and R.

   (c) Find the satellite's velocity at its apogee (the point where it is its farthest from the planet—here $3R$) in terms of G, M, and R.

(Last updated 4/20/2017)