

HW14Problem2

In[676]:= `ClearAll["Global`*"]`

In[677]:= `xcm = r[t] * Sin[theta1[t]] + (ell/2) * Sin[theta2[t]]`
`ycm = r[t] * Cos[theta1[t]] + (ell/2) * Cos[theta2[t]]`

Out[677]= $r[t] \sin[\theta_1[t]] + \frac{1}{2} \text{ell} \sin[\theta_2[t]]$

Out[678]= $\frac{1}{2} \text{ell} \cos[\theta_2[t]] + \cos[\theta_1[t]] r[t]$

In[679]:= `Tcm = (1/2) * M * (D[xcm, t]^2 + D[ycm, t]^2);`
`Trot = (1/2) * (M * ell^2 / 12) * theta2'[t]^2;`
`T = Simplify[Tcm + Trot]`

Out[681]= $\frac{1}{6} M (3 r'[t]^2 + 3 r[t]^2 \theta_1'[t]^2 + 3 \text{ell} \sin[\theta_1[t] - \theta_2[t]] r'[t] \theta_2'[t] + 3 \text{ell} \cos[\theta_1[t] - \theta_2[t]] r[t] \theta_1'[t] \theta_2'[t] + \text{ell}^2 \theta_2'[t]^2)$

In[682]:= `V = -M * g * ycm + (1/2) * k * (r[t] - B)^2;`

In[683]:= `L = Simplify[T - V]`

Out[683]= $-\frac{1}{2} k (B - r[t])^2 + g M \left(\frac{1}{2} \text{ell} \cos[\theta_2[t]] + \cos[\theta_1[t]] r[t] \right) + \frac{1}{6} M (3 r'[t]^2 + 3 r[t]^2 \theta_1'[t]^2 + 3 \text{ell} \sin[\theta_1[t] - \theta_2[t]] r'[t] \theta_2'[t] + 3 \text{ell} \cos[\theta_1[t] - \theta_2[t]] r[t] \theta_1'[t] \theta_2'[t] + \text{ell}^2 \theta_2'[t]^2)$

In[684]:= `pr = D[L, r'[t]];`
`Fr = D[L, r[t]];`
`zeror = Simplify[D[pr, t] - Fr]`

Out[686]= $r[t] (k - M \theta_1'[t]^2) + \frac{1}{2} (-2 B k - 2 g M \cos[\theta_1[t]] - \text{ell} M \cos[\theta_1[t] - \theta_2[t]] \theta_2'[t]^2 + 2 M r''[t] + \text{ell} M \sin[\theta_1[t] - \theta_2[t]] \theta_2''[t])$

In[687]:= `ptheta1 = D[L, theta1'[t]];`
`Ftheta1 = D[L, theta1[t]];`
`zertheta1 = Simplify[D[ptheta1, t] - Ftheta1]`

Out[689]= $\frac{1}{2} M r[t] (2 g \sin[\theta_1[t]] + 4 r'[t] \theta_1'[t] + \text{ell} \sin[\theta_1[t] - \theta_2[t]] \theta_2'[t]^2 + 2 r[t] \theta_1''[t] + \text{ell} \cos[\theta_1[t] - \theta_2[t]] \theta_2''[t])$

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In[690]:= ptheta2 = D[L, theta2'[t]];
Ftheta2 = D[L, theta2[t]];
zertheta2 = Simplify[D[ptheta2, t] - Ftheta2]
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Out[692]=  $\frac{1}{6} \text{ell} M \left( 3 g \text{Sin}[\text{theta2}[t]] + 6 \text{Cos}[\text{theta1}[t] - \text{theta2}[t]] r'[t] \text{theta1}'[t] + \right.$   

 $\left. 3 \text{Sin}[\text{theta1}[t] - \text{theta2}[t]] r''[t] + 3 r[t] \left( -\text{Sin}[\text{theta1}[t] - \text{theta2}[t]] \text{theta1}'[t]^2 + \right. \right.$   

 $\left. \left. \text{Cos}[\text{theta1}[t] - \text{theta2}[t]] \text{theta1}''[t] \right) + 2 \text{ell} \text{theta2}''[t] \right)$ 
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In[693]:= (* Simplify the three equations
(quantities that are zero) to compare with hand calculation *)
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In[694]:= Expand[zeror / M]
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Out[694]=  $-\frac{B k}{M} - g \text{Cos}[\text{theta1}[t]] + \frac{k r[t]}{M} - r[t] \text{theta1}'[t]^2 -$   

 $\frac{1}{2} \text{ell} \text{Cos}[\text{theta1}[t] - \text{theta2}[t]] \text{theta2}'[t]^2 +$   

 $r''[t] + \frac{1}{2} \text{ell} \text{Sin}[\text{theta1}[t] - \text{theta2}[t]] \text{theta2}''[t]$ 
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In[695]:= Expand[zertheta1 / M]
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Out[695]=  $g r[t] \text{Sin}[\text{theta1}[t]] + 2 r[t] r'[t] \text{theta1}'[t] +$   

 $\frac{1}{2} \text{ell} r[t] \text{Sin}[\text{theta1}[t] - \text{theta2}[t]] \text{theta2}'[t]^2 +$   

 $r[t]^2 \text{theta1}''[t] + \frac{1}{2} \text{ell} \text{Cos}[\text{theta1}[t] - \text{theta2}[t]] r[t] \text{theta2}''[t]$ 
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In[696]:= Expand[zertheta2 / M]
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Out[696]=  $\frac{1}{2} \text{ell} g \text{Sin}[\text{theta2}[t]] + \text{ell} \text{Cos}[\text{theta1}[t] - \text{theta2}[t]] r'[t] \text{theta1}'[t] -$   

 $\frac{1}{2} \text{ell} r[t] \text{Sin}[\text{theta1}[t] - \text{theta2}[t]] \text{theta1}'[t]^2 +$   

 $\frac{1}{2} \text{ell} \text{Sin}[\text{theta1}[t] - \text{theta2}[t]] r''[t] +$   

 $\frac{1}{2} \text{ell} \text{Cos}[\text{theta1}[t] - \text{theta2}[t]] r[t] \text{theta1}''[t] + \frac{1}{3} \text{ell}^2 \text{theta2}''[t]$ 
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