Phy 321 Spring 2017
HW 6.1

1.(a) $KE = \text{“}\frac{1}{2} I \omega^2\text{”} = \frac{1}{2}\left(\frac{MB^2}{3}\right)\dot{\theta}^2$

(b) $PE = \text{“}Mgh\text{”} = Mg\left(\frac{B}{2}\right)\sin(\theta)$

(c) $\frac{MB^2}{6}\dot{\theta}^2 + \frac{MgB}{2}\sin\theta = \frac{MgB}{2}$

so $\dot{\theta} = 0$ at $\theta = 90^\circ$

(d) $\frac{MB^2}{6} 2\dot{\theta}\ddot{\theta} + \frac{MgB}{2}\cos\theta\,\dot{\theta} = 0$

$$\Rightarrow \frac{MB^2}{3}\ddot{\theta} + \frac{MgB}{2}\cos\theta = 0$$

$$\boxed{\ddot{\theta} = \frac{-3}{2}\frac{g}{B}\cos\theta}$$

(e) center of mass is at $x_{cm} = \frac{B}{2}\cos\theta$

$$F_x = N_x = M\ddot{x}_{cm} = \frac{MB}{2}\frac{d^2}{dt^2}(\cos\theta)$$

$$= \frac{MB}{2}\frac{d}{dt}\left[-\sin\theta\,\dot{\theta}\right]$$

$$= \frac{-MB}{2}\left(\sin\theta\,\ddot{\theta} + \cos\theta\,\dot{\theta}^2\right)$$

From Energy Eq.,

$$\dot{\theta}^2 = \frac{6}{MB^2}\left(\frac{MgB}{2}\right)(1-\sin\theta)$$

also have

$$\ddot{\theta} = \frac{-3}{2}\,\frac{g}{B}\cos\theta$$

So

$$N_x = \frac{-MB}{2}\left[\frac{-3g}{2B}\cos\theta\sin\theta + \frac{3g}{B}(1-\sin\theta)\cos\theta\right]$$

$$= \left(\frac{-MB}{2}\right)\left(\frac{-3g}{2B}\right)\left[\cos\theta\sin\theta - 2(1-\sin\theta)\cos\theta\right]$$

$$\boxed{N_x = \left(\frac{3Mg}{4}\right)\cos\theta\,[3\sin\theta - 2]}$$

(f) center of mass is at $y_{cm} = \frac{B}{2}\sin\theta$

$$F_y = N_y - Mg = M\ddot{y}_{cm} = \frac{MB}{2}\frac{d^2}{dt^2}(\sin\theta)$$

$$\therefore\quad N_y = Mg + \frac{MB}{2}\frac{d^2}{dt^2}(\sin\theta)$$

$$N_y = Mg + \frac{MB}{2}\frac{d}{dt}\left[\cos\theta\ \dot{\theta}\right]$$

$$= Mg + \frac{MB}{2}\left[\cos\theta\ \ddot{\theta} - \sin\theta\ \dot{\theta}^2\right]$$

$$= Mg + \frac{MB}{2}\left[(\cos\theta)\left(\frac{-3g}{2B}\cos\theta\right) - \sin\theta\left(\frac{3g}{B}\right)(1-\sin\theta)\right]$$

$$= Mg + \left(\frac{MB}{2}\right)\left(\frac{-3g}{2B}\right)\left[\cos^2\theta + 2\sin\theta(1-\sin\theta)\right]$$

$$= Mg\left\{1 - \frac{3}{4}\left[\cos^2\theta + 2\sin\theta - 2\sin^2\theta\right]\right\}$$

$$= Mg\left\{1 - \frac{3}{4}\left[1-\sin^2\theta + 2\sin\theta - 2\sin^2\theta\right]\right\}$$

$$= Mg\left\{1 - \frac{3}{4}\left[1 + 2\sin\theta - 3\sin^2\theta\right]\right\}$$

$$= \frac{Mg}{4}\left\{1 - 6\sin\theta + 9\sin^2\theta\right\}$$

$$\boxed{N_y = \frac{Mg}{4}(1 - 3\sin\theta)^2}$$

(g) $L = \text{"}I\omega\text{"} = \pm \frac{MB^2}{3} \dot{\theta}$

(h) $\tau = \pm Mg \cdot \frac{B}{2} \cos\theta$

(i) $\tau = \dot{L}$ so $\frac{MgB}{2} \cos\theta = \pm \frac{MB^2}{3} \ddot{\theta}$

Correct sign is $\frac{MgB}{2} \cos\theta = -\frac{MB^2}{3} \ddot{\theta}$

(j) $\frac{N_x}{N_y} = \frac{3 \cos\theta (3 \sin\theta - 2)}{(1 - 3 \sin\theta)^2}$

Numerically, using Mathematica, in the region ~~B~~ $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, this has a single maximum of 0.3706 at $\theta = 0.9582 = 54.9°$

$\therefore$ need $\mu > 0.37$ to keep it from slipping before $45°$

Remark: $N_y = 0$ at $\sin\theta = \frac{1}{3}$

NOT REQUIRED

i.e. $\theta = 0.3398 = 19.5°$

so will always slip by there.

2. $U = a \sin(bxyz^2)$

$$F_x = -\frac{\partial U}{\partial x} = -a\cos(bxyz^2)\, byz^2$$

$$F_y = -\frac{\partial U}{\partial y} = -a\cos(bxyz^2)\, bxz^2$$

$$F_z = -\frac{\partial U}{\partial z} = -a\cos(bxyz^2)\, 2bxyz$$

at $x = y = z = 1$,

$$F_x = -ab\cos(b)$$

$$F_y = -ab\cos(b)$$

$$F_z = -2ab\cos(b)$$

(a) $$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= |ab\cos(b)|\sqrt{1+1+4}$$

$$= \sqrt{6}\,|ab\cos(b)|$$

HW 6.6

2(b) for $a > 0$ and $b = \pi$ at $x = y = z = 1$,

have $F_x = -a\pi \cos(\pi) = a\pi$

$F_y = a\pi$

$F_z = 2a\pi$

unit vector,

$F = a\pi\,(1, 1, 2)$

unitvector is $\dfrac{(1,1,2)}{\sqrt{1^2+1^2+2^2}}$

$= \left(\dfrac{1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}, \dfrac{2}{\sqrt{6}}\right)$

3. (a) $F_x = ax^2y^3 = -\dfrac{\partial U}{\partial x}$

at fixed y, $ax^2y^3 = -\dfrac{dU}{dx}$

$U = \dfrac{-ax^3y^3}{3} + \text{“const”}$

fcn of y

HW 6.7

$$\boxed{U = -\frac{a x^3 y^3}{3} + c(y)}$$

↑ arb. function

(3) (b) $F_y = -\frac{\partial U}{\partial y} = a x^3 y^2 + c'(y)$

Remark - not required -
could check

$$(\text{curl } F)_z = \frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x$$

$$= 3 a x^2 y^2 - 3 a x^2 y^2 = 0 \checkmark$$

(4)(a) $F = k(x, y, z)$

$$(\nabla \times F)_x = \frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y = 0$$

$$(\nabla \times F)_y = \frac{\partial}{\partial z} F_x - \frac{\partial}{\partial x} F_z = 0$$

$$(\nabla \times F)_z = \frac{\partial}{\partial x} F_y - \frac{\partial}{\partial y} F_x = 0$$

So $\boxed{\vec{\nabla} \times \vec{F} = 0}$

A quicker way is to notice That

$F = k\vec{r}$ makes $F = -\vec{\nabla} U$

where $U = \dfrac{-kr^2}{2}$

So F is conservative so curl F = 0

4.(b) $F = (Ax, By^2, Cz^3)$

$$(\nabla \times F)_x = \frac{\partial}{\partial y}(Cz^3) - \frac{\partial}{\partial z}(By^2) = 0$$

$$(\nabla \times F)_y = \frac{\partial}{\partial z}(Ax) - \frac{\partial}{\partial x}(Cz^3) = 0$$

$$(\nabla \times F)_z = \frac{\partial}{\partial x}(By^2) - \frac{\partial}{\partial y}(Ax) = 0$$

so again ~~curl~~ $\boxed{(\vec{\nabla} \times \vec{F}) = 0}$

Again a quick way is to notice

$\vec{F} = -\vec{\nabla} U$ where

$$U = \frac{-Ax^2}{2} - \frac{By^3}{3} - \frac{Cz^4}{4}$$

4. (c) $F = (Ay^2, Bx, Cz)$

$$(\nabla \times F)_x = \frac{\partial}{\partial y}(Cz) - \frac{\partial}{\partial z}(Bx) = 0$$

$$(\nabla \times F)_y = \frac{\partial}{\partial z}(Ay^2) - \frac{\partial}{\partial x}(Cz) = 0$$

$$(\nabla \times F)_z = \frac{\partial}{\partial x}(Bx) - \frac{\partial}{\partial y}(Ay^2)$$

$$= B - 2Ay$$

So $\boxed{\vec{\nabla} \times \vec{F} = (B - 2Ay)\,\hat{z}}$