Phy 321 Spring 2017
HW 9.1

1. (a) $F(t) = 1 - \frac{2\omega|t|}{\pi}$ $\quad -\frac{\pi}{\omega} < t < \frac{\pi}{\omega}$

$$F(t) = \sum A_n e^{in\omega t}$$

$$\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} F(t) e^{-im\omega t} dt = \sum A_n \int_{-\frac{\pi}{\omega}}^{\pi/\omega} e^{i(n-m)\omega t} dt$$

$$= A_m \frac{2\pi}{\omega}$$

$$A_n = \frac{\omega}{2\pi} \int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} F(t) e^{-in\omega t} dt$$

$$= \left(\frac{\omega}{2\pi}\right) \left\{ \int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} e^{-in\omega t} dt \right.$$

$$+ \int_0^{\pi/\omega} \left(\frac{-2\omega t}{\pi}\right) e^{-in\omega t} dt$$

$$\left. + \int_{-\pi/\omega}^{0} \left(\frac{2\omega t}{\pi}\right) e^{-in\omega t} dt \right\}$$

Let $t = -u$ in 3^{rd} integral,

then change u to t

$$A_n = \left(\frac{\omega}{2\pi}\right)\left\{\left[\frac{e^{-in\omega t}}{-in\omega}\right]_{-\pi/\omega}^{\pi/\omega}\right.$$

$$+\int_0^{\pi/\omega}\left(\frac{-2\omega t}{\pi}\right)e^{-in\omega t}\,dt$$

$$\left.+\int_0^{\pi/\omega}\left(\frac{-2\omega t}{\pi}\right)e^{in\omega t}\,dt\right\}$$

$$= \left(\frac{\omega}{2\pi}\right)\left\{\underbrace{\left(\frac{e^{-in\pi}-e^{in\pi}}{-in\omega}\right)}_{0}\right.$$

$$\left.-\frac{2\omega}{\pi}\int_0^{\pi/\omega} t\,dt\left(e^{-in\omega t}+e^{+in\omega t}\right)\right\}$$

HW 9.3

$$A_n = -\left(\frac{\omega}{\pi}\right)^2 (2) \int_0^{\pi/\omega} t \cos(n\omega t)\, dt$$

$$u = t \qquad dv = \cos(n\omega t)\, dt$$

$$du = dt \qquad v = \frac{\sin(n\omega t)}{n\omega}$$

$$A_n = -2\left(\frac{\omega}{\pi}\right)^2 \left\{ \left[t \frac{\sin(n\omega t)}{n\omega} \right]_0^{\pi/\omega} - \int_0^{\pi/\omega} dt \frac{\sin(n\omega t)}{n\omega} \right\}$$

$$= -2\left(\frac{\omega}{\pi}\right)^2 \left\{ \underbrace{\left(\frac{\pi}{\omega}\right) \frac{\sin(n\pi)}{n\omega}}_{0} - \frac{1}{n\omega} \left[\frac{\cos(n\omega t)}{-n\omega} \right]_0^{\pi/\omega} \right\}$$

$$= \frac{2\omega}{n\pi^2} \frac{1}{-n\omega} \left[\cos(n\pi) - 1 \right]$$

$$\boxed{A_n = \left(\frac{2}{\pi^2}\right) \frac{1}{n^2} \left[1 - (-1)^n \right]}$$

can also write as

$$A_n = \begin{cases} \dfrac{4}{\pi^2 n^2} & \text{if } n \text{ odd} \\ 0 & \text{if } n \text{ even} \end{cases}$$

$$F(t) = \sum_{n=-\infty}^{\infty} A_n e^{in\omega t}$$

$$= \sum_{n \text{ odd}, n>0} \frac{4}{\pi^2 n^2} \left(e^{in\omega t} + e^{-in\omega t} \right)$$

$$\boxed{F(t) = \left(\frac{8}{\pi^2}\right) \sum_{n=1,3,5,7,\dots} \frac{1}{n^2} \cos(n\omega t)}$$

(b) $$F = \left(\frac{8}{\pi^2}\right) \sum_{\substack{n \text{ odd} \\ -\infty}}^{\infty} \frac{1}{n^2} \left(\frac{e^{in\omega t} + e^{-in\omega t}}{2} \right)$$

$$x = \left(\frac{4}{\pi^2}\right) \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{1}{n^2} \left\{ \frac{e^{in\omega t}}{-\omega^2 + 2\beta i n\omega + \omega_0^2} + \frac{e^{-in\omega t}}{-\omega^2 - 2\beta i n\omega + \omega_0^2} \right\}$$

HW 9.5

(c) $x = \frac{4}{\pi^2} \sum_{\substack{n\ \text{odd} \\ n=-\infty}}^{\infty} \frac{1}{n^2} \Bigg\{$

$$\frac{(\cos n\omega t + i \sin n\omega t)(-\omega^2 - 2\beta i n\omega + \omega_0^2)}{(-\omega^2 + \omega_0^2)^2 + (2\beta n\omega)^2}$$

$$+ \frac{(\cos n\omega t - i \sin n\omega t)(-\omega^2 + 2\beta i n\omega + \omega_0^2)}{(-\omega^2 + \omega_0^2)^2 + (2\beta n\omega)^2} \Bigg\}$$

$$= \left(\frac{4}{\pi^2}\right) \sum_{\substack{n\ \text{odd} \\ n=-\infty}}^{\infty} \left(\frac{1}{n^2}\right) \frac{1}{(-\omega^2 + \omega_0^2)^2 + (2\beta n\omega)^2}$$

$$\Big\{ \cos(n\omega t)(-\omega^2 + \omega_0^2)(2)$$

$$+ \sin(n\omega t)(2\beta n\omega)(2) \Big\}$$

HW 9.6

$$x = \left(\frac{16}{\pi^2}\right) \sum_{n=1,3,5,\ldots} \frac{1}{n^2} \frac{1}{(-\omega^{2n}+\omega_0^2)^2 + (2\beta n\omega)^2}$$

$$\left\{ (-\omega^2+\omega_0^2)\cos(n\omega t) + (2\beta n\omega)\sin(n\omega t) \right\}$$

$$\cancel{(d)\ x = \left(\frac{16}{\pi^2}\right)\left\{ \frac{\left(-\frac{\omega_0^2}{9}+\omega_0^2\right)\cos\frac{\omega_0 t}{3}}{\left(-\frac{\omega_0^2}{9}+\omega_0^2\right)^2 + \left(\frac{2\beta\omega_0}{3}\right)^2} \right.}$$

$$(d)\quad x = \frac{16}{\pi^2}\left\{ \frac{\left(\frac{8}{9}\omega_0^2\right)\cos\left(\frac{\omega_0 t}{3}\right) + \frac{2\beta\omega_0}{3}\sin\left(\frac{\omega_0 t}{3}\right)}{\left(\frac{8}{9}\omega_0^2\right)^2 + \left(\frac{2\beta\omega_0}{3}\right)^2} \right.$$

$$\left. + \frac{2\beta\omega_0 \sin(\omega_0 t)}{(2\beta\omega_0)^2} \right\}$$

For small β, ↑ dominates.