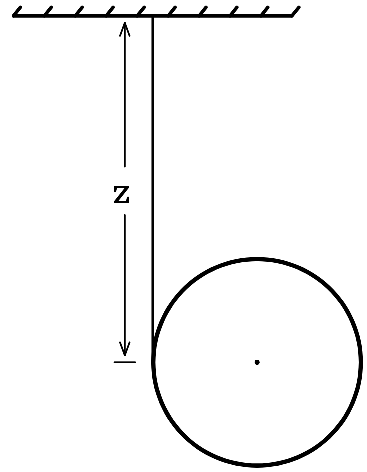
Physics 422/820 – Fall 2015

Homework #1, Due at beginning of class Friday Sept 11.



1. [5 pts] A thin flexible string is wrapped around a uniform homogeneous cylinder of radius R and mass M. The free end of the string is tied to the ceiling and the cylinder is allowed to fall starting from rest with the string vertical with $z = z_0$. The string does not slip on the cylinder, so the cylinder rotates as the string unwinds from it.

   (a) The string will remain vertical. Explain why.

   (b) Use conservation of energy to find the motion: i.e., find $z(t)$ as a function of M, R, z_0 and g.

   (c) Find the tension in the string.

2. [5 pts] A uniform chain of length ℓ lies pushed together at the edge of a table, with length x_0 hanging over the edge (Textbook problem 1.6). It is released from rest at time $t = 0$.

   (a) Find the differential equation of motion using Newton's law: $F = M\,\ddot{x}_{\rm cm}$, where $x_{\rm cm}$ is the position of the center of mass.

   (b) Find the differential equation of motion using Newton's law in the form Newton wrote it: $F = \dot{p}$, where p is the momentum. Check that this agrees with your previous result.

   (c) Show that the equation given for v^2 in the textbook satisfies your equation of motion.

3. [10 pts] Consider the integral

$$\mathcal{I} = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 + 4y^2 \right] dx$$

   (a) Find the smooth function $y(x)$ that minimizes this integral subject to the boundary conditions $y(0) = 0$ and $y(1) = 1$.

   (b) Find the minimum value of $\mathcal{I}$ that is achieved by your solution for $y(x)$.

   (c) Find the smallest value of $\mathcal{I}$ that can be obtained by a quadratic polynomial $y = c_0 + c_1 x + c_2 x^2$ that satisfies the same boundary conditions. (Because $y(0) = 0$ and $y(1) = 1$, you can write the polynomial as $y = x + c\,x(1 - x)$, where c is the only free parameter.) You probably want to use Mathematica or some other computer method to avoid doing a bunch of algebra by hand.

   (d) Find the smallest value of $\mathcal{I}$ that can be obtained by a cubic polynomial $y = x + x(1 - x)(c_1 + c_2 x)$ which also satisfies the $y(0) = 0$ and $y(1) = 1$ same boundary conditions. *You certainly want to use Mathematica or some other computer method for this—life is too short to do calculations like that by hand; and if symbolic computer methods are difficult for you, you just need more practice at them!*

(Last updated 9/2/2015.)