Physics 422/820 – Fall 2015

Homework #2, Due at beginning of class Friday Sept 18.

1. [6 pts] When we solved the Brachistochrone problem in class (this problem was first solved more than 300 years ago) our result was

$$
\begin{aligned}
x &= \frac{y_0}{2}(\psi - \sin\psi) \\
y &= \frac{y_0}{2}(1 - \cos\psi) \\
\psi &= \sqrt{\frac{2g}{y_0}}\, t
\end{aligned}
$$

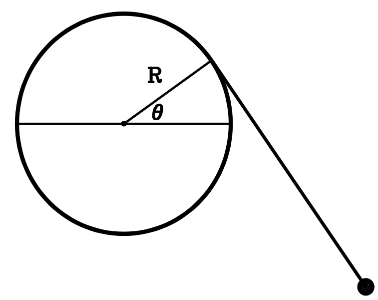
   The mass M starts from rest at $x = 0$, $y = 0$. Assume that it ends at $x = A$, $y = A/4$.

   (a) Find the travel time.

   (b) Find the average speed.

   (c) Find the magnitude of the force supplied by the ramp as a function of time.

2. [6 pts] A thin flexible string is wrapped around a fixed cylinder of radius R. One end of the string is attached at the top of the cylinder, so it cannot slip. The other end of the string is tied to a small mass M. The length of the string is ℓ. (The part of the string that is in contact with the cylinder clearly has length $(\pi/2 - \theta)R$; the straight portion of the string is ℓ minus that.) Hint: to solve this problem, it is convenient to find the position of the mass in Cartesian coordinates (x, y) as a function of θ.



   (a) Write the kinetic and potential energy as a function of θ, M, g, R.

   (b) Write the Lagrangian and use it to obtain a second order differential equation of motion for θ.

   (c) Use conservation of energy to obtain a first order differential equation of motion for θ.

3. [8 pts] A point particle of mass M slides without friction on the surface of a cone with half-angle A. The axis of the cone points upward, so in Cartesian coordinates the gravitational force points in the $-\hat{z}$ direction and the cone is defined by

$$\sqrt{x^2 + y^2} = z \tan A\,.$$

   (a) Write the Lagrangian using cylindrical coordinates, which are defined by

$$x = \rho\cos\phi\,, \quad y = \rho\sin\phi\,, \quad z = z$$

   (b) Use that Lagrangian to obtain the differential equations of motion.
   (You do not need to solve those the equations.)

   (c) Write the Lagrangian using spherical coordinates, which are defined by

$$x = r\sin\theta\cos\phi\,, \quad y = r\sin\theta\sin\phi\,, \quad z = r\cos\theta$$

   (d) Use that Lagrangian to obtain the differential equations of motion.
   (You do not need to solve those equations.)

(Last updated 9/11/2015.)