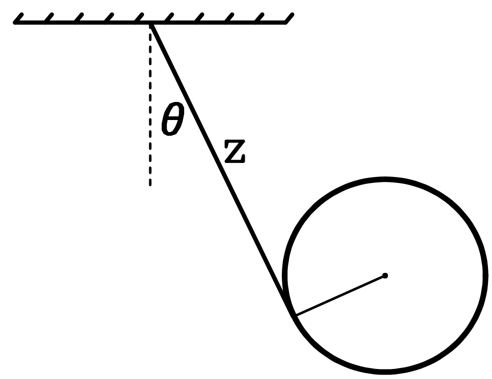
Physics 422/820 – Fall 2015

Homework #3, Due at beginning of class Friday Sept 25.

1. [7 pts] A thin flexible string is wrapped many times around a cylinder of radius R, mass M, and moment of inertia $\mathcal{I}$. The string cannot slip on the cylinder, but it can unwind freely. One end of the string is tied to the ceiling and the cylinder is allowed to fall starting from rest at some initial angle θ_0 and initial unwrapped string length $z = z_0$.



   (a) Find the Lagrangian using the coordinates z and θ.

   (b) Use your Lagrangian to find the differential equations of motion.

   (c) Write Newton's law to relate the acceleration of the center of mass to the tension in the string. Also write Newton's law for angular acceleration, which will give you one more equation involving the tension. Use that relation to eliminate the tension variable in your equations from the acceleration of the center of mass. This will result in two differential equations of motion.

   (d) Show that the two equations you found in part (c) are equivalent to the two equations you found in part (b).

2. [7 pts] A point particle of mass M slides without friction on the surface of a cone with half-angle A. The axis of the cone points upward, i.e., the pointy end of the cone points downward. Hence in Cartesian coordinates the gravitational force points in the $-\hat{z}$ direction and the cone is defined by

$$\sqrt{x^2 + y^2} = z \tan A \, .$$

   (a) Write the Lagrangian using coordinates defined by

$$x = \rho \cos\phi \, , \quad y = \rho \sin\phi \, , \quad z = z$$

   Use z and ϕ as the actual coordinates, with ρ determined by the requirement that the particle stays on the cone.

   (b) Use one of the two Lagrange equations of motion together with the energy conservation equation to write an "effective energy" equation that relates $\dot{z}$ to z, with no reference to ϕ or $\dot{\phi}$.

   (c) Find the velocity required to cause motion in a circular orbit at a fixed value $z = z_0$, and check that your solution agrees with what you learned in first-year physics about centripetal force.

   (d) Consider motions where z remains close to z_0, by approximating the "effective potential" as a parabola near its minimum. The motion in z will then correspond to a harmonic oscillator. Express the angular frequency of oscillation in $z - z_0$ in terms of g and z_0.

   (e) Express the angular frequency of the rotation in ϕ in terms of g and z_0. Find the cone half-angle A that makes this equal to your result from part (d), which produces a closed orbit.

3. [6 pts] A point particle of mass M moving in one dimension has kinetic energy $\frac{1}{2} M \dot{x}^2$. If the potential energy is $U(x) = a x^2 + b|x|$, find the period of the oscillations. Give your answer as a function of the maximum displacement x_0 and a, b, M. The turning points are x_0 and $-x_0$. (This is problem 2.6 in the textbook, but the answer given there is not correct, and it is expressed in terms of energy rather than x_0.)

(Last updated 9/19/2015.)