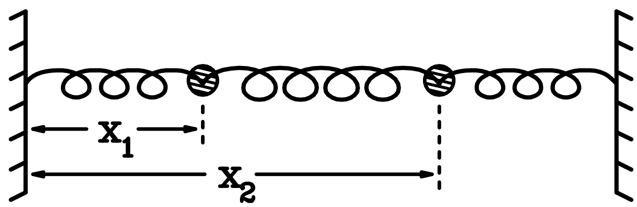
Physics 422/820 – Fall 2015

Homework #4, Due at beginning of class Friday Oct 2.

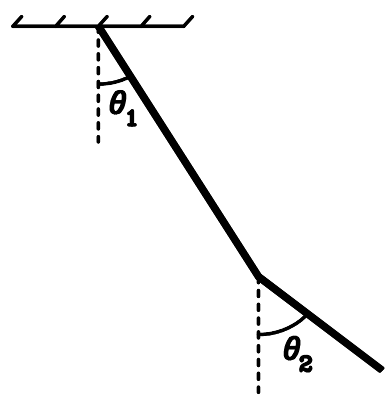
1. [6 pts] Two particles of equal mass M are attached to the junctions of three springs as shown in the figure. The masses move only in the horizontal direction. The two springs attached to walls each have spring constant K, while the middle spring has spring constant $3\,K$. All three springs have unstretched length a. Begin your work using the coordinates x_1 and x_2 that are defined in the figure, but define new coordinates q_1 and q_2 that differ from those by constants chosen to make $q_1 = q_2 = 0$ correspond to the equilibrium position. (You can make any assumption you like about the distance between the two walls: the results won't depend on it.)

   (a) Find the eigenfrequencies and normal modes of the system.

   (b) Determine the particle positions as a function of time if at time $t = 0$, both masses are at their equilibrium positions and particle 1 has velocity v and particle 2 has velocity 0.



2. [8 pts] A uniform stick of mass M_1 and length L_1 hangs from the ceiling. A second uniform stick of mass M_2 and length L_2 hangs from the lower end of the first stick to form a double pendulum. The motion is confined to a vertical plane, so the only coordinates are θ_1 and θ_2.



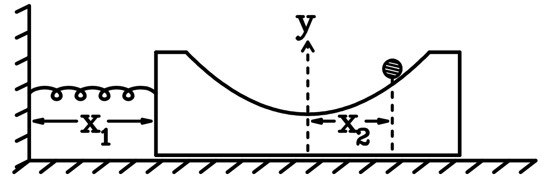
   (a) Write the Lagrangian without assuming small angles.

   (b) Find the quadratic equation for ω^2 that determines the two possible angular frequencies of oscillation ω_1 and ω_2 in the limit of small oscillations.

   (c) Solve the quadratic equation for ω_1 and ω_2 in the case $L_2 = L_1$ and $M_2 = 0.3M_1$. Express your answer in terms of M_1, L_1 and g. (You can thank me for choosing the length and mass ratios to make the solutions for ω_1 and ω_2 somewhat simple.)

   (d) Make a sketch of what the system looks like when oscillating in each of the two normal modes.

3. [6 pts] A block of mass M_1 slides without friction on a horizontal surface. It is attached to a wall by a spring with spring constant K, whose unstretched length C is long enough that the block never hits the wall. The upper surface of the block has a channel that is cut in the shape of a parabola: $y = A + B\,x_2^2$. A point mass M_2 slides without friction along that parabola.



   (a) Write the Lagrangian using the coordinates x_1 and x_2, without making small-oscillation approximations.

   (b) Find the normal mode frequencies for small oscillations in the special case $M_1 = M_2$.

(Last updated 9/27/2015.)