Physics 422/820 – Fall 2015

Homework #5, Due at beginning of class Friday Oct 9.

1. N particles of equal mass M slide freely on a circle. They are connected by N springs with equal spring constant K. Use coordinates $q_1, \ldots, q_N$ which are measured from the equilibrium positions, so you don't care about the unstretched lengths. Feel free to use the natural units $M = 1$ and $K = 1$. You may prefer to do the $N = 6$ case first, because its greater symmetry makes it easier than the $N = 5$ case. But both problems can be solved by thinking about what the solutions must look like, rather than using brute force with the T and V matrices.

   Several of the normal modes here involve degeneracy; i.e., two or more of the ω_i values can be equal to each other. Another way to say that, is that there can be more than one independent amplitude vector $A^{(i)}$ corresponding to some particular mode frequency ω. When that happens, each amplitude vector can be replaced by a linear combination of all of the amplitude vectors for that frequency.

   To obtain unique answers to these problems in the case of degeneracy, use the linear-combination freedom to make at least one of the elements of each $A^{(i)}$ equal to 0 whenever you can.

   (a) [10 pts] Find the natural mode frequencies ω_i for $i = 1, \ldots, N$ and their corresponding amplitudes $(A_1^{(i)}, \ldots, A_N^{(i)})$ for $N = 5$.

   (b) [10 pts] Find the natural mode frequencies ω_i for $i = 1, \ldots, N$ and their corresponding amplitudes $(A_1^{(i)}, \ldots, A_N^{(i)})$ for $N = 6$.

(Last updated 10/6/2015.)