Physics 422/820 – Fall 2015

Homework #6, Due at beginning of class Friday Oct 16.

1. [5 pts] A solid cone has height H and circular base of radius R. It has mass M, but its mass distribution is not uniform: using cylindrical coordinates with the z direction along the symmetry axis of the cone, with the pointy end of the cone at $z = 0$ and the circular base at $z = H$, the mass per unit volume is proportional to z. Find its principal moments of inertia about its center of mass. *(Hint: first find the moments of inertia about the pointy end, which makes the integrals simple; then use the parallel axis theorem.)*

2. [5 pts] Find the principal moments of inertia about the center of mass for a solid hemisphere (half of a sphere—e.g., the half north of the equator) with a uniform mass distribution. Do this problem in two ways:
   (a) The dumb way: just do the integrals using spherical coordinates.
   (b) The smart way: use your knowledge of the moments of inertia of a full sphere and the principal axis theorem. (You still have to do one integral to locate the center of mass.)

3. [5 pts] A thin uniform sheet of metal in the shape of an equilateral triangle hangs from the ceiling by one of its corners. It is free to swing in the plane of the triangle. Find the frequency of small oscillations in terms of g and the length S of the side of the triangle.

4. [5 pts] Three point masses are rigidly attached to each other by massless rods:

$$\begin{aligned} m_1 &= 1\,M \quad \text{at} \quad (x, y, z) = (-B, B, 0) \\ m_2 &= 2\,M \quad \text{at} \quad (x, y, z) = (B, 0, B) \\ m_3 &= 3\,M \quad \text{at} \quad (x, y, z) = (B, B, -B)\,. \end{aligned}$$

   (a) Find the inertia tensor for rotations about $(x, y, z) = (0, 0, 0)$.
   (b) Find the principal moments of inertia for rotations about $(x, y, z) = (0, 0, 0)$.
   (c) Find the principal axes for rotations about $(x, y, z) = (0, 0, 0)$.

(Last updated 10/9/2015.)