Physics 422/820 – Fall 2015

Homework #7, Due at beginning of class Friday Oct 23.

1. [6 pts] A homogeneous cube with sides of length S is balanced on one edge. Given an infinitessimal push, it falls over onto one side. Assuming that the edge in contact with the table never slips, find $\dot{\theta}$ as a function of θ by energy conervation.

2. [7 pts] In a particular coordinate frame, the moment of inertia tensor of a rigid body is

$$\mathbf{I} = \begin{pmatrix} 3 & 4 & 0 \\ 4 & 9 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

in some units. The instantaneous angular velocity vector in that frame is given by $\omega = (2, 3, 4)$ in some units.

   (a) Find the principal moments of inertia.

   (b) Find a rotation matrix a that transforms to a new coordinate system in which the moment of inertia tensor is diagonal.

   (c) Find the moment of inertia tensor $\mathbf{I}'$ and the angular velocity vector ω' in the new coordinate system.

   (d) Compute the kinetic energy and the magnitude of the angular momentum in both frames, and compare the results.

3. [7 pts] The matrix a is orthogonal

$$a = \begin{pmatrix} 0.0637 & -0.7944 & 0.6040 \\ 0.9448 & 0.2429 & 0.2198 \\ -0.3214 & 0.5567 & 0.7660 \end{pmatrix}$$

   (a) Find ϕ, θ, ψ to write this in the form $a = R_z(\phi)\, R_y(\theta)\, R_z(\psi)$. (Hint: a good way to do this is to examine $\hat{e}_3 \cdot a \cdot \hat{e}_3$ first; then $\hat{e}_1 \cdot a \cdot \hat{e}_3$ and $\hat{e}_2 \cdot a \cdot \hat{e}_3$; then ... you get the idea.

   (b) Find the axis that a rotates about. (Hint: the rotation doesn't do anything to its axis, so the axis must be an eigenvector with the obvious eigenvalue.)

   (c) Find the magnitude of the rotation angle in degrees. (Hint: You could choose an arbitrary vector that is perpendicular to the rotation axis and use the matrix a to rotate that vector; then examine the dot product between the original vector and its rotated form. Or you could use a trick based on looking at the trace of the matrix a.)

(Last updated 10/16/2015.)