Physics 422/820 – Fall 2015

Homework #8, Due at beginning of class Friday Oct 30.

1. [5 pts] A uniform cylindrical metal rod has mass M, length A, and radius R. A second identical rod is attached at right angles to the first rod at its center to form a **T** shape. The end of the second rod just touches the symmetry axis of the first rod. For simplicity, assume that the density in the small region where the two rods overlap is equal to twice the density in each rod: that way, you can find the complete inertia tensor by just adding the conrtributions from the two rods, without worrying about any effect due to drilling a hole in the first rod to make room for the second.

   Find the three principal moments of inertia for the combined system in the center of mass frame, assuming $R = A/6$.

2. [5 pts] An object quite similar to the **T** of the previous problem has principal moments of inertia equal to 3, 9, and 7. It rotates freely in zero gravity. Its angular velocity vector is $(\omega_1, \omega_2, \omega_3)$ in the body frame.

   (a) Use the conserved quantities T (= kinetic energy = total energy) and $\vec{L}^2$ (magnitude of angular momentum squared) to solve for ω_1 and ω_2 as a function of ω_3. To determine T and $\vec{L}^2$, assume that the angular velocity vector in the body frame is $(\omega_{10}, \omega_{20}, \omega_{30})$ at $t = 0$.

   (b) Plug your results from (a) into the Euler equation of motion $I_3\,\dot{\omega}_3 = (I_1 - I_2)\,\omega_1\,\omega_2$ to obtain an equation that relates $\dot{\omega}_3$ to ω_3. In doing this, let the ω_3 axis correspond to the middle moment of inertia, for which the motion is known to be unstable.

   (c) Suppose that $(\omega_{10}, \omega_{20}, \omega_{30}) = (0.001,\ 0.001,\ 1.0)$ in some units. Show that the system will spend most of its time with ω_3 close to 1 or -1 with ω_1 and ω_2 small.

3. [5 pts] Model a dinner plate as a two-dimensional disk of uniform density. Suppose the plate is rotating freely (in the International Space Station, for example, where gravitational torques can be neglected.) The symmetry axis of the plate makes a constant angle of 30° with respect to the $\hat{z}$ direction in space, which corresponds to the direction of the angular momentum vector. The symmetry axis precesses around the angular momentum at angular velocity Ω. Hence two of the Euler angles are given by $\theta = \pi/6$ and $\phi = \Omega\, t$. The third Euler angle is given by $\psi = C\, t$.

   (a) Find the value of the constant C that is required to satisfy the Euler equations of motion.

   (b) Find the three components of the angular momentum vector in the Body frame.

   (c) Apply the rotation $R_z(\phi)\, R_y(\theta)\, R_z(\psi)$ to your result from part (b), to find the components of the angular momentum vector in the Space frame, and check that your result makes sense.

   (d) The vector (1, 0, 0) in the Body frame corresponds to a radial line painted on the plate. Transform that vector to the Space frame, and compare its observed rate of rotation (when projected onto the plane perpendicular to the angular momentum) with the rate Ω of the wobbling of the plate. *This problem is mentioned by Feynman in Surely, You're Joking ..., though he remembers a factor of two in the wrong direction. Our version of this problem is tougher than Feynman's, because he took the $\theta \to 0$ limit.*

4. [5 pts] A projectile is fired in the vertical direction at latitude 60°. It reaches a maximum height H. Calculate its displacement in the east-west direction when it returns to ground level due to the Coriolis force. Neglect air resistance.

(Last updated 10/26/2015.)