Physics 422/820 – Fall 2015

Homework #10, Due at beginning of class Friday Nov 13.

1. [8 pts] A particle of mass M is moving under the influence of a central force

$$F(r) = -\frac{\alpha}{r^2} - \frac{\beta}{r^4}$$

where $\alpha > 0$ and $\beta > 0$. It has angular momentum ℓ.

   (a) Find the potential $V(R)$ and write the kinetic energy using polar coordinates.

   (b) Find the minimum value of ℓ for which circular orbits ($r = \text{const}$) are possible.

   (c) Sketch $V_{\text{eff}}(r)$ for the three possible cases: (1) where no circular orbits are possible; (2) where only one circular orbit is possible; and (3) where more than one circular orbit is possible.

   (d) If only one circular orbit is possible, is it stable or unstable? (Explain why.)

2. [8 pts] (P. W. Johnson problem 6.1) A particle moves under the influence of a central potential $V(r)$ where $V(r) \to 0$ in the limit $r \to \infty$.

   (a) Show that if the particle moves along the curve $r^2 = a^2 \cos(2\,\theta)$ then the potential is given by $V(r) = -\ell^2\, a^4/(2\, M\, r^6)$.

   (b) Find the velocity of the particle as a function of r.

   (c) Find the time taken for the particle to travel from $r = a$ to $r = 0$.

   (Johnson gives answers for parts (b) and (c), but one or both of those answers might be wrong.)

3. [4 pts] A particle of mass M moves under the influence of a repulsive spherically symmetric force defined by the potential

$$V(r) = \frac{A}{r^2}$$

where $A > 0$. Find r as a function of time if the total energy is E and the angular momentum is ℓ. (E and ℓ are of course constants of the motion.)

(Last updated 11/6/2015.)