Physics 422/820 – Fall 2015

Homework #11, Due at beginning of class Friday Nov 20.

1. [5 pts] Consider the motion of a charged particle in a time-independent magnetic field. The Hamiltonian is

$$H = \frac{1}{2M} (\vec{p} - e\vec{A})^2 .$$

   (a) Find the Lagrangian as a function of its correct variables $\vec{r}$ and $\dot{\vec{r}}$. (This was worked out in lecture.)

   (b) Let the magnetic field $\vec{B}$ be given by the field due to a long straight wire carrying a current i. You know its form from first-year physics. Find the corresponding magnetic vector potential $\vec{A}$, which is defined by $\vec{B} = \vec{\nabla} \times \vec{A}$. By making a convenient choice of gauge, you can assume $\vec{A}$ points in the $\hat{z}$ direction, where $\hat{z}$ points along the direction of the current.

   (c) Find the Lagrangian for this field, using the cylindrical coordinates $x = r\cos\phi$, $y = r\sin\phi$, $z = z$.

   (d) Use this new Lagrangian to find the Hamiltonian as a function of its correct variables.

   (e) Your Hamiltonian gives you three independent constants of the motion. Use those constants to write an Effective Potential equation which relates $\dot{r}$ to a function of r.

2. [5 pts] (Based on an incorrect problem in Goldstein.) The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\dot{y}/x + f y^2 \dot{x}\dot{z} + g\dot{y}^2 + k\sqrt{x^2 + y^2} ,$$

   where a, b, f, g and k are constants.

   (a) Find the Hamiltonian.

   (b) Write the Hamiltonian equations of motion; but you do not need to solve them.

   (c) What are the conserved quantities? (Note that the Hamiltonian gives you one more conserved quantity than the Lagrangian does.)

3. [5 pts] (Based on a problem in Goldstein.) The Lagrangian for a system with one degree of freedom is

$$L = \frac{M}{2} \left(\dot{q}^2 \sin^2 \omega t + \dot{q}\, q\, \omega \sin 2\omega t + q^2 \omega^2\right) ,$$

   where M and ω are constants

   (a) Find the Hamiltonian as a function of its correct variables.

   (b) Is this Hamiltonian conserved? (State why or why not.)

   (c) Find the new Hamiltonian that is obtained by introducing a new coordinate defined by $Q = q\sin\omega t$. (You will first need to find the new Lagrangian.)

   (d) Use the new Hamiltonian to solve for $Q(t)$ and hence find the most general solution $q(t)$ for the motion according to the original Lagrangian.

4. [5 pts] (Based on an old MSU Subject Exam problem.) A particle of mass M slides without friction inside the surface of a frictionless paraboloid of revolution $z = A(x^2 + y^2)$ where $A > 0$ is a constant. The symmetry axis z of the paraboloid is vertical, so there is a gravitational force in the $-\hat{z}$ direction. Use r and ϕ as generalized coordinates, where $x = r\cos\phi$ and $y = r\sin\phi$.

   (a) Find the Hamiltonian and use it to find the Hamiltonian equations of motion.

   (b) How many constants of the motion did you find?

(Last updated 11/14/2015.)