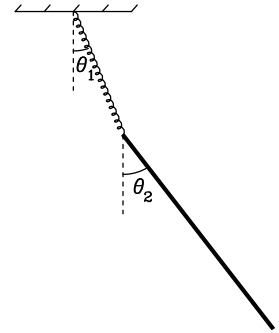Based on a problem from Classical Mechanics Subject Exam (August 29, 2016)

A uniform thin rod of length b and mass M hangs from the ceiling by a massless spring that has spring constant k and unstretched length r_0.

1. Calculate the potential energy of the system (due to gravity and the spring) as a function of θ_1, θ_2, and $r =$ length of the spring.

2. Calculate the kinetic energy of the system as a function of θ_1, $\dot{\theta}_1$, θ_2, $\dot{\theta}_2$, r, and $\dot{r}$. (Hint: The moment of inertia of the rod about its center of mass is $Mb^2/12$.)

3. Use the Lagrangian to find the equations of motion.

4. Find the motion in the limit of small oscillations.