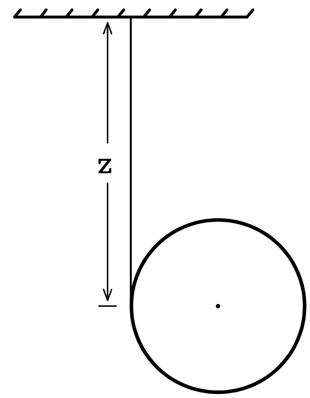
Physics 422/820 – Fall 2016

Homework #1, Due at beginning of class Friday Sept 9.

1. [5 pts] A thin flexible string is wrapped around a uniform homogeneous solid cylinder of radius R and mass M. The free end of the string is tied to the ceiling and the cylinder is allowed to fall starting from rest with the string vertical with $z = z_0$. The string does not slip on the cylinder, so the cylinder rotates as the string unwinds from it.



   (a) The string will remain vertical. Explain why.

   (b) Use conservation of energy to find the motion: i.e., find $z(t)$ as a function of M, R, z_0 and g.

   (c) Find the tension in the string.

2. [5 pts] A uniform chain of length ℓ lies pushed together at the edge of a table, with length x_0 hanging over the edge. It is released from rest at time $t = 0$.

   (a) Find the differential equation of motion using Newton's law: $F = M\,\ddot{x}_{\rm cm}$, where $x_{\rm cm}$ is the position of the center of mass of the entire chain, and F is the total force on the entire chain.

   (b) Find the differential equation of motion using Newton's law in the form Newton wrote it: $F = \dot{p}$, where p is the momentum. Check that this agrees with your previous result.

   (c) It is possible to solve the equation of motion in the special case $x_0 = 0$. **This is not required; but you might enjoy doing it.** (Hint: try to let x to be proportional to a power of t.)

   (d) It is possible to solve the equation of motion in the general case in the form of an elliptic integral. **This is not required; but if you can do it, you get a round of applause from the class.** (Hint: find a differential equation for the square of the velocity $v = \dot{x}$ as a function of x by using $d(v^2)/dt = d(v^2)/dx \times dx/dt = v\,d(v^2)/dx$.)

3. [10 pts] Consider the integral

$$\mathcal{I} = \int_0^1 \left[\left(\frac{dy}{dx} \right)^2 + 4y^2 \right] dx$$

   (a) Find the smooth function $y(x)$ that minimizes this integral subject to the boundary conditions $y(0) = 0$ and $y(1) = 1$.

   (b) Find the minimum value of $\mathcal{I}$ that is achieved by your solution for $y(x)$.

   (c) Find the smallest value of $\mathcal{I}$ that can be obtained by a quadratic polynomial $y = c_0 + c_1 x + c_2 x^2$ that satisfies the same boundary conditions. (Because $y(0) = 0$ and $y(1) = 1$, you can write the polynomial as $y = x + c\,x(1-x)$, where c is the only free parameter.) You probably want to use Mathematica or some other computer method to avoid doing a bunch of algebra by hand.

   (d) Find the smallest value of $\mathcal{I}$ that can be obtained by a cubic polynomial $y = x + x(1-x)(c_1 + c_2 x)$ which also satisfies the $y(0) = 0$ and $y(1) = 1$ same boundary conditions. *You certainly want to use Mathematica or some other computer method for this—life is too short to do calculations like that by hand; and if symbolic computer methods are difficult for you, you just need more practice at them!*

(Last updated 9/8/2016.)