Physics 422/820 – Fall 2016

Homework #2, Due at beginning of class Friday Sept 16.

1. [6 pts] When we solved the Brachistochrone problem in class (this problem was first solved more than 300 years ago) our result was

$$\begin{aligned} x &= A(\theta - \sin\theta) \\ y &= A(1 - \cos\theta) \\ \theta &= \sqrt{\frac{g}{A}}\, t \end{aligned}$$

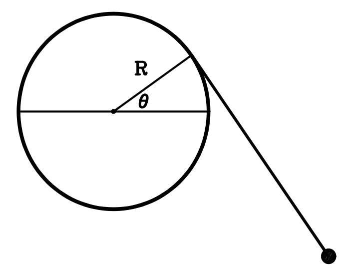
The mass M starts from rest at $x = 0$, $y = 0$. Assume that it ends at $x = B$, $y = B/5$.

   (a) Find the travel time.

   (b) Find the average speed along the track. (I.e., find the distance traveled along the track and divide by your answer to part (a).)

   (c) Find the force supplied by the track as a function of θ.

2. [6 pts] A thin flexible string is wrapped around a fixed (not moving and not rotating) cylinder of radius R. One end of the string is attached at the top of the cylinder, so it cannot slip. The other end of the string is tied to a small mass M. The length of the string is ℓ. (The part of the string that is in contact with the cylinder clearly has length $(\pi/2 - \theta)R$; the straight portion of the string is ℓ minus that.) Hint: to solve this problem, it is convenient to find the position of the mass in Cartesian coordinates (x, y) as a function of θ.



   (a) Write the kinetic and potential energy as a function of θ, M, g, R.

   (b) Write the Lagrangian and use it to obtain a second order differential equation of motion for θ.

   (c) Use conservation of energy to obtain a first order differential equation of motion for θ.

   (d) Show that your answers to (b) and (c) are consistent with each other.

3. [8 pts] A point particle of mass M slides without friction on the surface of a cone with half-angle A. The axis of the cone points upward, i.e., the pointy end of the cone points downward. Hence in Cartesian coordinates the gravitational force points in the $-\hat{z}$ direction and the cone is defined by $\sqrt{x^2 + y^2} = z \tan A$.

   (a) Write the Lagrangian using cylindrical coordinates, which are defined by

$$x = \rho\cos\phi\,, \quad y = \rho\sin\phi\,, \quad z = z$$

   Use z and ϕ as the coordinates; then ρ is determined by the requirement that the particle stays on the cone.

   (b) Use the Lagrange equations of motion to recognize the constant of motion that corresponds to angular momentum.

   (c) Use the Lagrange equations of motion to prove that the total energy $E = T + V$ is constant.

   (d) Use the constants of energy and angular momentum to obtain an "Effective Energy" equation that relates z to $\dot{z}$.

(Last updated 9/9/2016.)