Physics 422/820 – Fall 2016

Homework #3, Due at beginning of class Friday Sept 23.

1. [8 pts] Consider the damped driven pendulum equation

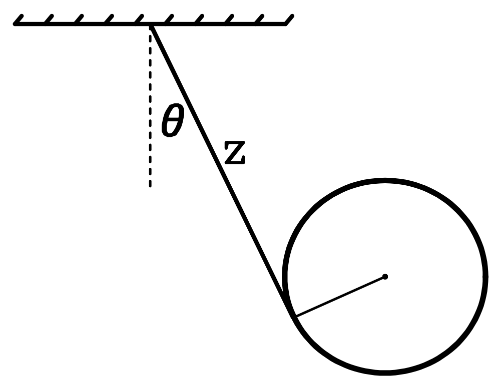
$$\ddot{x} + 2b\dot{x} + \omega_0^2 \sin x = \gamma\omega_0^2 \sin(\omega t)$$

Let $\omega = 2\pi$ so that you are measuring time in units of the period of the driving oscillator. Let $\omega_0 = (3/2)\omega$ and $b = \omega_0/4$. (These values are the same as used in the textbook examples, but the equation is slightly different.)

   (a) Solve the equation numerically with initial conditions $x = 0$ and $\dot{x} = 0$ at $t = 0$ for $\gamma = 1.082$. Show that after a sufficient time, the motion becomes periodic. Compare the frequency of that motion with the frequency of the driving term.

   (b) Repeat part (a) with $\gamma = 1.083$. The dramatic change in the behavior of the system caused by a small change in the equation is an example of chaotic behavior.

2. [6 pts] A thin flexible string is wrapped many times around a hollow cylinder of radius R, mass M, and moment of inertia $M R^2$. The string cannot slip on the cylinder, but it can unwind freely. One end of the string is tied to the ceiling and the cylinder is allowed to fall starting from rest at some initial angle θ_0 and initial unwrapped string length $z = z_0$.



   (a) Find the Lagrangian using the coordinates z and θ.

   (b) Use your Lagrangian to find the differential equations of motion.

   (c) Write Newton's law to relate the acceleration of the center of mass to the tension in the string. Also write Newton's law for angular acceleration, which will give you one more equation involving the tension. Use that relation to eliminate the tension variable in your equations from the acceleration of the center of mass. This will result in two differential equations of motion.

   (d) Show that the two equations you found in part (c) are equivalent to the two equations you found in part (b).

3. [6 pts] A point particle of mass M moving in one dimension has kinetic energy $\frac{1}{2} M \dot{x}^2$. If the potential energy is $U(x) = a\,x^2 + b\,|x|$ where $a > 0$ and $b > 0$, find the period of the oscillations. Give your answer as a function of the maximum displacement x_0 and a, b, M. The turning points are x_0 and $-x_0$.

(Last updated 9/20/2016.)