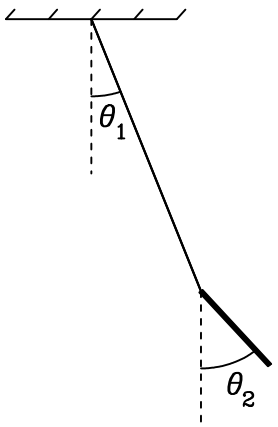
Physics 422/820 – Fall 2016

Homework #4, due at beginning of class Friday Sept 30.

1. [12 pts] A uniform stick of mass M and length b hangs from the ceiling by a massless thread of length ℓ. The motion is confined to a vertical plane, so the only coordinates are θ_1 and θ_2.



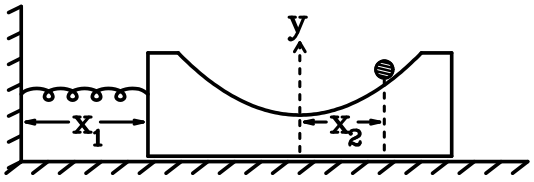
   (a) In the limit of small oscillations, find the ratio of the two normal mode frequencies as a function of the dimensionless ratio b/ℓ.

   (b) Solve the small-oscillation equation of motion for the case $b = \ell/4$, with initial conditions $\theta_2(0) = 2°$, $\theta_1(0) = \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$ and use your result to make a graph of $\theta_1(t)$.

   (c) Make a graph of $\theta_1(t)$ for the initial conditions of part (b) by solving the exact equation of motion numerically via NDSolve. (Your result should be nearly the same as what you found in part (b) because the small-angle approximation should be good.)

   (d) Make a graph of $\theta_1(t)$ for initial conditions that are the same as in part (b) except that $\theta_2(0) = 129.82°$. Because this angle is so large, you will obviously have to use the method of part (c).

2. [8 pts] A block of mass M_1 slides without friction on a horizontal surface. It is attached to a wall by a spring with spring constant K, whose unstretched length C is long enough that the block never hits the wall. The upper surface of the block has a channel that is cut in the shape of a parabola: $y = A + B\,x_2^2$. A point mass M_2 slides without friction along that parabola.



   (a) Write the Lagrangian using the coordinates x_1 and x_2, without making small-oscillation approximations.

   (b) Find the normal mode frequencies for small oscillations in the special case $M_1 = M_2$.

(Last updated 9/23/2016.)