## Physics 422/820 - Fall 2016

## Homework \#5, Due at beginning of class Friday Oct 7 .

1. [20 pts] Two particles of equal mass $M$ are attached to the junctions of three springs as shown in the figure. The masses move only in the horizontal direction. The two springs attached to walls each have spring constant $K$, while the middle spring has spring constant $3 K$. The springs all have unstretched length $a$, and the distance between the walls is $b$.
(a) Write the Lagrangian for this system as a function of $x_{1}, x_{2}, \dot{x}_{1}, \dot{x}_{2}$.
(b) Find the equilibrium (non-oscillating) positions for $x_{1}$ and $x_{2}$.
(c) Introduce new coordinates $q_{1}=x_{1}-$ const $_{1}$ and $q_{2}=x_{2}$ - const ${ }_{2}$, where the constant shifts are chosen to make $q_{1}=q_{2}=0$ the equilibrium configuration; and find the Lagrangian in these nice coordinates.
(d) Find the normal mode frequencies and their corresponding amplitude vectors using the nice coordinates.
(e) Determine the particle positions as a function of time if at time $t=0$, both masses are at their equilibrium positions and particle 1 has velocity $v$ and particle 2 has velocity 0 .

