Physics 422/820 – Fall 2016

Homework #6, Due at beginning of class Friday Oct 14.

1. [10 pts] N particles, where N is even, slide freely on a circle. They are connected by N springs of equal spring constant K which connect each neighboring pair of particles. The masses alternate between two values: $M_1 = M_3 = \ldots = M_{N-1}$ and $M_2 = M_4 = \ldots = M_N$.

   Use coordinates $q_1, \ldots, q_N$ that are measured from the equilibrium positions, so you don't care about the unstretched lengths. Feel free to use the natural unit $K = 1$.

   Find the normal mode frequencies ω_i for $i = 1, \ldots, N$ and their corresponding amplitudes $(A_1^{(i)}, \ldots, A_N^{(i)})$ vectors **for the case N=6, with M$_1$=M$_3$=M$_5$=3, and M$_2$=M$_4$=M$_6$=8.**

   I can think of three ways to do this problem:

   (a) Most elegantly: solve the problem for arbitrary even integer N, by solving a difference equation using the method shown in lecture, and then set $N = 6$ in your answer;

   (b) Use symmetry arguments and orthogonality (relative to the Kinetic Energy matrix) arguments to work out the normal modes one at a time;

   (c) Directly use the Lagrangian method.

   For full credit, you must use at least two of these methods, and check that you get the same answer both ways.

2. [5 pts] A solid cone has height H and circular base of radius R. It has mass M, but its mass distribution is not uniform: using cylindrical coordinates with the z direction along the symmetry axis of the cone, with the pointy end of the cone at $z = 0$ and the circular base at $z = H$, the mass per unit volume is proportional to z. Find its principal moments of inertia about its center of mass. *(Hint: first find the moments of inertia about the pointy end, which makes the integrals simple; then use the parallel axis theorem.)*

3. [5 pts] Find the principal moments of inertia about the center of mass for a solid hemisphere (half of a sphere—e.g., the half north of the equator) with a uniform mass distribution. Do this problem in two ways:

   (a) The dumb way: just do the integrals using spherical coordinates.

   (b) The smart way: use your knowledge of the moments of inertia of a full sphere and the parallel axis theorem. (You still have to do one integral to locate the center of mass.)

(Last updated 10/13/2016.)