Physics 422/820 – Fall 2016

Homework #7, Due at beginning of class Friday Oct 21.

1. [8 pts] A homogeneous cube with sides of length S is balanced on one edge. Given an infinitessimal push, it falls over onto one side.

   (a) Assuming that the edge in contact with the table never slips, find $\dot{\theta}$ as a function of θ by energy conservation.

   (b) Find the minimum coefficient of friction that is necessary to keep it from slipping.

2. [6 pts] Three point masses are rigidly attached to each other by massless rods:

$$\begin{aligned} m_1 &= 1\,M \quad \text{at} \quad (x,y,z) = (-B, B, 0) \\ m_2 &= 2\,M \quad \text{at} \quad (x,y,z) = (B, 0, B) \\ m_3 &= 3\,M \quad \text{at} \quad (x,y,z) = (B, B, -B)\ . \end{aligned}$$

   (a) Find the inertia tensor for rotations about the point $(x, y, z) = (0, 0, 0)$.

   (b) Find the principal moments of inertia for rotations about the point $(x, y, z) = (0, 0, 0)$.

   (c) Find the principal axes for rotations about the point $(x, y, z) = (0, 0, 0)$.

3. [6 pts] In a particular coordinate frame, the moment of inertia tensor of a rigid body is

$$\mathbf{I} = \begin{pmatrix} 3 & 4 & 0 \\ 4 & 9 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

   in some units. The instantaneous angular velocity vector in that frame is given by $\omega = (2, 3, 4)$ in some units.

   (a) Find the principal moments of inertia.

   (b) Find a rotation matrix a that transforms to a new coordinate system in which the moment of inertia tensor is diagonal.

   (c) Find the moment of inertia tensor $\mathbf{I}'$ and the angular velocity vector ω' in the new coordinate system.

   (d) Compute the kinetic energy and the magnitude of the angular momentum in both frames, and compare the results.

(Last updated 10/14/2016.)