Physics 422/820 – Fall 2016

Homework #10, Due at beginning of class Friday Nov 11.

1. [20 pts] An artillery shell is fired at initial speed v_0 in the direction Southwest (i.e., bearing 225°) at an angle of 45° above the horizontal, by a warship at rest at latitude 45° South. Find the distance between the place where the shell hits the water, and the place where it would be expected to hit if you ignored the Coriolis force. Ignore air resistance and assume the shell is fired from water level. Approximate the earth as flat, but include the Coriolis effect.

   Do the calculation to first order in ω = rotation rate of the earth. One way to do it is by the following recipe:

   (a) Define coordinate directions, e.g., $\hat{x}$=north, $\hat{y}$=west, $\hat{z}$=up.

   (b) Write the equations of motion, which will be of the form

   $$(\ddot{x}, \ddot{y}, \ddot{z}) \;=\; (\text{Gravity term}) \;+\; (\text{Coriolis term})$$

   (c) Set $\omega = 0$ on the right-hand side of that equation and solve for $x(t)$, $y(t)$, $z(t)$, imposing the initial conditions $((x(0), y(0), z(0)) = (0, 0, 0)$ and the given $\dot{x}(0), \dot{y}(0), \dot{z}(0)$ values. This gives the motion in the limit $\omega \to 0$.

   (d) Put your $\omega \to 0$ solution into the right-hand side of the equations of motion. This will give you equations of motion that are accurate to order ω^1. Solve those equations, again imposing the known initial conditions on position and velocity at time zero.

   (e) Find the time the projectile strikes the water by solving the equation $z = 0$. This time will involve ω. You can simplify it by expanding it in powers of ω and dropping terms higher than ω^1.

   (f) Find the place that the projectile hits the water by substituting the time that z reaches zero into your solutions for x and y. *You can simplify those results by expanding them in powers of ω and dropping terms higher than ω^1—which makes sense because your solutions are only valid to that order anyway.*

   (g) The displacement caused by the Coriolis force is now easy to find (to first order in ω) by comparing the point $(x, y, 0)$ where it hits with the $\omega \to 0$ limit of that quantity.

   Extra challenge (not required): find the displacement to accuracy ω^2 .

(Last updated 11/7/2016.)