

Physics 422/820 – Fall 2016

Homework #11, Due at beginning of class Friday Nov 18.

1. [5 pts] Consider the motion of a charged particle in a time-independent magnetic field. The Hamiltonian is

$$H = \frac{1}{2M} (\vec{p} - e\vec{A})^2 .$$

- Find the Lagrangian as a function of its correct variables \vec{r} and $\dot{\vec{r}}$. (This was worked out in lecture.)
- Let the magnetic field \vec{B} be given by the field due to a long straight wire carrying a steady current i . You know its form from first-year physics. Find a corresponding magnetic vector potential \vec{A} , which is defined by $\vec{B} = \vec{\nabla} \times \vec{A}$. To do that, you can assume \vec{A} points in the \hat{z} direction, where \hat{z} points along the direction of the current—this corresponds to choosing a specific gauge.
- Find the Lagrangian for this field, using the cylindrical coordinates $x = r \cos \phi$, $y = r \sin \phi$, $z = z$.
- Use this new Lagrangian to find the Hamiltonian as a function of its correct variables.
- Your Hamiltonian gives you three independent constants of the motion. Use those constants to write an Effective Potential equation that relates \dot{r} to a function of r .

2. [5 pts] (Based on an incorrect problem in Goldstein.) The Lagrangian for a system can be written as

$$L = a\dot{x}^2 + b\dot{y}/x + f y^2 \dot{x}\dot{z} + g\dot{y}^2 + k\sqrt{x^2 + y^2} ,$$

where a , b , f , g and k are constants.

- Find the Hamiltonian.
 - Write the Hamiltonian equations of motion; but you do not need to solve them.
 - What are the conserved quantities? (Note that the Hamiltonian gives you one more conserved quantity than the Lagrangian does.)
3. [5 pts] (Based on a problem in Goldstein.) The Lagrangian for a system with one degree of freedom is

$$L = \frac{M}{2} (\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2 \omega^2) ,$$

where M and ω are constants

- Find the Hamiltonian as a function of its correct variables.
 - Is this Hamiltonian conserved? (State why or why not.)
 - Find the new Hamiltonian that is obtained by introducing a new coordinate defined by $Q = q \sin \omega t$. (You will first need to find the new Lagrangian.)
 - Use the new Hamiltonian to solve for $Q(t)$ and hence find the most general solution $q(t)$ for the motion according to the original Lagrangian.
4. [5 pts] (Based on an old MSU Subject Exam problem.) A particle of mass M slides without friction inside the surface of a frictionless paraboloid of revolution $z = A(x^2 + y^2)$ where $A > 0$ is a constant. The symmetry axis z of the paraboloid is vertical, so there is a gravitational force in the $-\hat{z}$ direction. Use r and ϕ as generalized coordinates, where $x = r \cos \phi$ and $y = r \sin \phi$.
- Find the Hamiltonian and use it to find the Hamiltonian equations of motion.
 - How many constants of the motion did you find?