StringMassCylinder2

In[2224]:=
```
Clear["Global`*"]
```

In[2917]:=
```
r = R * {Cos[theta[t]], Sin[theta[t]]} + u * {Sin[theta[t]], -Cos[theta[t]]}
```

Out[2917]=
$$\{R \text{Cos}[\text{theta}[t]] + u \text{Sin}[\text{theta}[t]], -u \text{Cos}[\text{theta}[t]] + R \text{Sin}[\text{theta}[t]]\}$$

In[2918]:=
```
u = ell - R * (Pi / 2 - theta[t])
```

Out[2918]=
$$\text{ell} - R \left(\frac{\pi}{2} - \text{theta}[t]\right)$$

In[2919]:=
```
Solve[u == 0, theta[t]]
theta[t] /. %[[1]]
```

Out[2919]=
$$\left\{\left\{\text{theta}[t] \to \frac{-2\,\text{ell} + \pi R}{2 R}\right\}\right\}$$

Out[2920]=
$$\frac{-2\,\text{ell} + \pi R}{2 R}$$

In[2921]:=
```
Solve[theta0 == %, ell]
ell /. %[[1]]
```

Out[2921]=
$$\left\{\left\{\text{ell} \to \frac{1}{2} R (\pi - 2\,\text{theta0})\right\}\right\}$$

Out[2922]=
$$\frac{1}{2} R (\pi - 2\,\text{theta0})$$

In[2923]:=
```
ell = %
```

Out[2923]=
$$\frac{1}{2} R (\pi - 2\,\text{theta0})$$

In[2925]:=
```
rr = Simplify[r]
```

Out[2925]=
$$\{R (\text{Cos}[\text{theta}[t]] - \text{theta0}\, \text{Sin}[\text{theta}[t]] + \text{Sin}[\text{theta}[t]]\, \text{theta}[t]),$$
$$R (\text{theta0}\, \text{Cos}[\text{theta}[t]] + \text{Sin}[\text{theta}[t]] - \text{Cos}[\text{theta}[t]]\, \text{theta}[t])\}$$

In[2926]:=
```
v = D[rr, t]
```

Out[2926]=
$$\{R (-\text{theta0}\, \text{Cos}[\text{theta}[t]]\, \text{theta}'[t] + \text{Cos}[\text{theta}[t]]\, \text{theta}[t]\, \text{theta}'[t]),$$
$$R (-\text{theta0}\, \text{Sin}[\text{theta}[t]]\, \text{theta}'[t] + \text{Sin}[\text{theta}[t]]\, \text{theta}[t]\, \text{theta}'[t])\}$$

In[2929]:=
```
T = Simplify[(1 / 2) * M * v.v]
```

Out[2929]=
$$\frac{1}{2} M R^2 (\text{theta0} - \text{theta}[t])^2\, \text{theta}'[t]^2$$

In[2977]:=
```
V = Simplify[M * g * r.{0, 1}]
```

Out[2977]=
$$g M R (\text{theta0}\, \text{Cos}[\text{theta}[t]] + \text{Sin}[\text{theta}[t]] - \text{Cos}[\text{theta}[t]]\, \text{theta}[t])$$

In[2978]:=
```
L = T - V;
```

In[2979]:=
```
p = D[L, theta'[t]]
```

Out[2979]=
$$M R^2 (\text{theta0} - \text{theta}[t])^2 \text{theta}'[t]$$

In[2980]:=
```
f = Simplify[D[L, theta[t]]]
```

Out[2980]=
$$-M R (\text{theta0} - \text{theta}[t]) \left(-g \text{Sin}[\text{theta}[t]] + R \text{theta}'[t]^2\right)$$

In[2981]:=
```
zero1 = Simplify[D[p, t] - f]
```

Out[2981]=
$$M R (\text{theta0} - \text{theta}[t]) \left(-g \text{Sin}[\text{theta}[t]] - R \text{theta}'[t]^2 + R (\text{theta0} - \text{theta}[t]) \text{theta}''[t]\right)$$

In[2982]:=
```
zero2 = zero1 / (M * R * (theta0 - theta[t]))
```

Out[2982]=
$$-g \text{Sin}[\text{theta}[t]] - R \text{theta}'[t]^2 + R (\text{theta0} - \text{theta}[t]) \text{theta}''[t]$$

In[2983]:=
```
zero3 = zero2 /. {g → 1, R → 1}
```

Out[2983]=
$$-\text{Sin}[\text{theta}[t]] - \text{theta}'[t]^2 + (\text{theta0} - \text{theta}[t]) \text{theta}''[t]$$

In[2984]:=
```
a = Simplify[D[v, t]]
```

Out[2984]=
$$\{R \left((\text{Cos}[\text{theta}[t]] + \text{theta0} \text{Sin}[\text{theta}[t]] - \text{Sin}[\text{theta}[t]] \text{theta}[t]) \text{theta}'[t]^2 + \text{Cos}[\text{theta}[t]] (-\text{theta0} + \text{theta}[t]) \text{theta}''[t]\right),$$
$$R \left((-\text{theta0} \text{Cos}[\text{theta}[t]] + \text{Sin}[\text{theta}[t]] + \text{Cos}[\text{theta}[t]] \text{theta}[t]) \text{theta}'[t]^2 + \text{Sin}[\text{theta}[t]] (-\text{theta0} + \text{theta}[t]) \text{theta}''[t]\right)\}$$

In[2985]:=
```
sol1 = Solve[zero3 == 0, theta''[t]]
```

Out[2985]=
$$\{\{\text{theta}''[t] \to \left(\text{Sin}[\text{theta}[t]] + \text{theta}'[t]^2\right) / (\text{theta0} - \text{theta}[t])\}\}$$

In[2986]:=
```
aa = Simplify[a /. sol1[[1]]]
```

Out[2986]=
$$\{-R \text{Sin}[\text{theta}[t]] \left(\text{Cos}[\text{theta}[t]] + (-\text{theta0} + \text{theta}[t]) \text{theta}'[t]^2\right),$$
$$-R \left(\text{Sin}[\text{theta}[t]]^2 + \text{Cos}[\text{theta}[t]] (\text{theta0} - \text{theta}[t]) \text{theta}'[t]^2\right)\}$$

In[2987]:=
```
T
```

Out[2987]=
$$\frac{1}{2} M R^2 (\text{theta0} - \text{theta}[t])^2 \text{theta}'[t]^2$$

In[2988]:=
```
V
```

Out[2988]=
$$g M R (\text{theta0} \text{Cos}[\text{theta}[t]] + \text{Sin}[\text{theta}[t]] - \text{Cos}[\text{theta}[t]] \text{theta}[t])$$

In[2989]:=
```
sol2 = Solve[T + V == (V /. theta[t] → thetaS), theta'[t]]
```

Out[2989]=
```
{{theta'[t] →
    -((Sqrt[2] Sqrt[g] Sqrt[(theta0 Cos[thetaS] - thetaS Cos[thetaS] - theta0 Cos[theta[t]] +
            Sin[thetaS] - Sin[theta[t]] + Cos[theta[t]] theta[t])]) /
      (Sqrt[R theta0^2 - 2 R theta0 theta[t] + R theta[t]^2]))}, {theta'[t] →
   (Sqrt[2] Sqrt[g] Sqrt[(theta0 Cos[thetaS] - thetaS Cos[thetaS] - theta0 Cos[theta[t]] +
          Sin[thetaS] - Sin[theta[t]] + Cos[theta[t]] theta[t])]) /
    (Sqrt[R theta0^2 - 2 R theta0 theta[t] + R theta[t]^2])}}
```

In[2990]:=
```
aa
```

Out[2990]=
```
{-R Sin[theta[t]] (Cos[theta[t]] + (-theta0 + theta[t]) theta'[t]^2),
 -R (Sin[theta[t]]^2 + Cos[theta[t]] (theta0 - theta[t]) theta'[t]^2)}
```

In[2991]:=
```
aaa = Simplify[(aa /. sol2[[1]]) /. {g → 1, R → 1}]
```

Out[2991]=
```
{(Sin[theta[t]] (2 (theta0 - thetaS) Cos[thetaS] - 3 theta0 Cos[theta[t]] +
       2 Sin[thetaS] - 2 Sin[theta[t]] + 3 Cos[theta[t]] theta[t])) /
   (theta0 - theta[t]), -Sin[theta[t]]^2 +
   (2 Cos[theta[t]] ((-theta0 + thetaS) Cos[thetaS] + theta0 Cos[theta[t]] -
        Sin[thetaS] + Sin[theta[t]] - Cos[theta[t]] theta[t])) / (theta0 - theta[t])}
```

In[2992]:=
```
TensionVec = Simplify[aaa + {0, 1}]
```

Out[2992]=
```
{(Sin[theta[t]] (2 (theta0 - thetaS) Cos[thetaS] - 3 theta0 Cos[theta[t]] +
       2 Sin[thetaS] - 2 Sin[theta[t]] + 3 Cos[theta[t]] theta[t])) / (theta0 - theta[t]),
  (Cos[theta[t]] (2 (theta0 - thetaS) Cos[thetaS] - 3 theta0 Cos[theta[t]] + 2 Sin[
       thetaS] - 2 Sin[theta[t]] + 3 Cos[theta[t]] theta[t])) / (-theta0 + theta[t])}
```

In[2993]:=
```
StringDir = {Sin[theta[t]], -Cos[theta[t]]}
```

Out[2993]=
```
{Sin[theta[t]], -Cos[theta[t]]}
```

In[2994]:=
```
tension = Simplify[-TensionVec.StringDir]
```

Out[2994]=
```
(-2 (theta0 - thetaS) Cos[thetaS] + 3 theta0 Cos[theta[t]] - 2 Sin[thetaS] +
   2 Sin[theta[t]] - 3 Cos[theta[t]] theta[t]) / (theta0 - theta[t])
```

In[2995]:=
```
rDir = {Cos[theta[t]], Sin[theta[t]]}
```

Out[2995]=
```
{Cos[theta[t]], Sin[theta[t]]}
```

In[2996]:=
```
zerocheck = Simplify[-TensionVec.rDir]
```

Out[2996]=
```
0
```

In[3056]:=
```
tmax = 20;
th0 = -91 * Pi / 180;
thS = -90 * Pi / 180;
nd = NDSolve[{(zero3 /. theta0 → th0) == 0, theta'[0] == 0, theta[0] == thS},
   theta[t], {t, 0, tmax}];
Plot[theta[t] /. nd[[1]], {t, 0, tmax}]
Plot[(tension /. {theta0 → th0, thetaS → thS}) /. nd[[1]], {t, 0, tmax}]
```

In[3082]:=
```
L
```

Out[3082]=
$$-g\,M\,R\,(\text{theta0}\,\text{Cos}[\text{theta}[t]]+\text{Sin}[\text{theta}[t]]-\text{Cos}[\text{theta}[t]]\,\text{theta}[t])+\frac{1}{2}\,M\,R^2\,(\text{theta0}-\text{theta}[t])^2\,\text{theta}'[t]^2$$

In[3083]:=
```
LL = L /. {g → 1, M → 1, R → 1}
```

Out[3083]=
$$-\text{theta0}\,\text{Cos}[\text{theta}[t]]-\text{Sin}[\text{theta}[t]]+\text{Cos}[\text{theta}[t]]\,\text{theta}[t]+\frac{1}{2}\,(\text{theta0}-\text{theta}[t])^2\,\text{theta}'[t]^2$$

In[3084]:=
```
LLL = LL /. theta'[t] → thdot[t]
```

Out[3084]=
$$-\text{theta0}\,\text{Cos}[\text{theta}[t]]-\text{Sin}[\text{theta}[t]]+\frac{1}{2}\,\text{thdot}[t]^2\,(\text{theta0}-\text{theta}[t])^2+\text{Cos}[\text{theta}[t]]\,\text{theta}[t]$$

In[3085]:=
```
pp = D[LLL, thdot[t]]
```

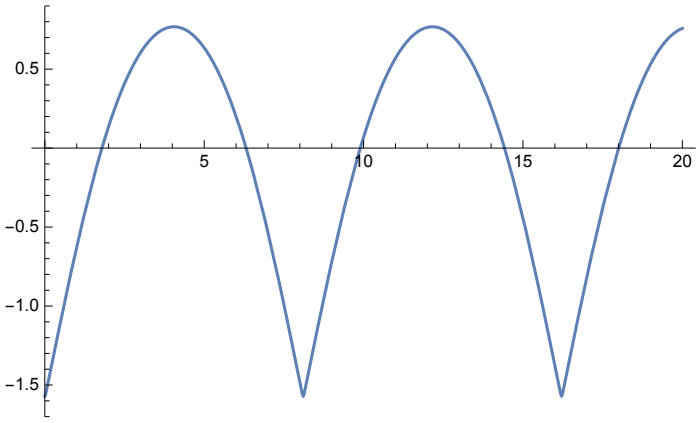
Out[3085]=
$$\text{thdot}[t]\,(\text{theta0}-\text{theta}[t])^2$$

In[3086]:=
```
ff = D[LLL, theta[t]]
```

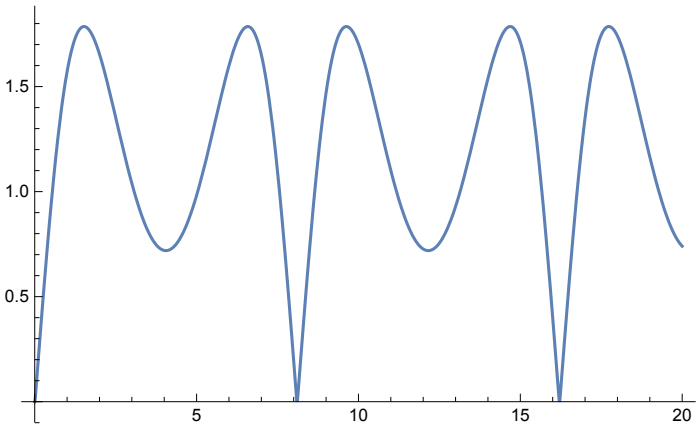
Out[3086]=
$$\text{theta0}\,\text{Sin}[\text{theta}[t]]-\text{thdot}[t]^2\,(\text{theta0}-\text{theta}[t])-\text{Sin}[\text{theta}[t]]\,\text{theta}[t]$$

In[3094]:=
```
tmax = 20;
th0 = -91 * Pi / 180;
thS = -90 * Pi / 180;
nd = NDSolve[{theta'[t] == thdot[t], 0 == (D[pp, t] - ff) /. theta0 → th0,
    thdot[0] == 0, theta[0] == thS}, {theta[t], thdot[t]}, {t, 0, tmax}];
Plot[theta[t] /. nd[[1]], {t, 0, tmax}]
Plot[(tension /. {theta0 → th0, thetaS → thS}) /. nd[[1]], {t, 0, tmax}]
```

Out[3098]=

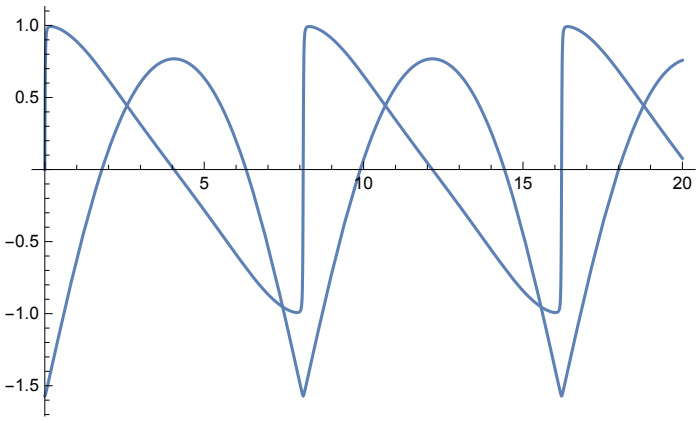


Out[3099]=



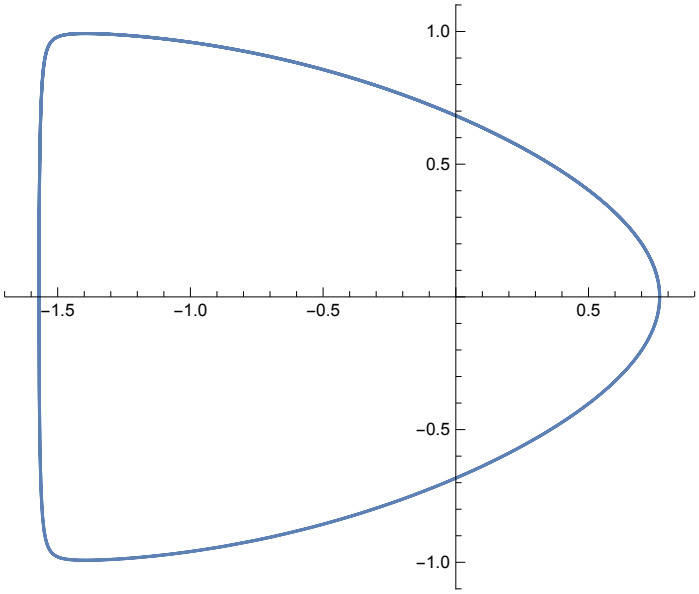
In[3112]:=
```
Plot[{theta[t], thdot[t]} /. nd[[1]], {t, 0, tmax}]
```

Out[3112]=



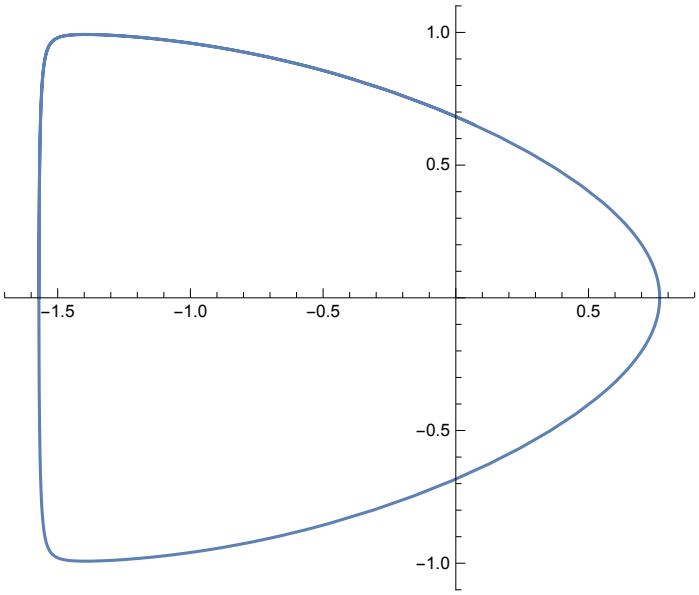
In[3113]:=
```
ParametricPlot[{theta[t], thdot[t]} /. nd[[1]], {t, 0, tmax}]
```

Out[3113]=



In[3134]:=
```
tmax = 10;
th0 = -91 * Pi / 180;
thS = -90 * Pi / 180;
nd = NDSolve[{theta'[t] == thdot[t], 0 == (D[pp, t] - ff) /. theta0 → th0,
    thdot[0] == 0, theta[0] == thS}, {theta[t], thdot[t]}, {t, 0, tmax}];
ParametricPlot[{theta[t], thdot[t]} /. nd[[1]], {t, 0, tmax}]
```

Out[3138]=



In[3154]:=
```
tmax = 10;
th0 = -91 * Pi / 180;
thS = -90 * Pi / 180;
nd = NDSolve[{theta'[t] == thdot[t], 0 == (D[pp, t] - ff) /. theta0 → th0,
    thdot[0] == 0, theta[0] == thS}, {theta[t], thdot[t]}, {t, 0, tmax}];
plot90 = ParametricPlot[{theta[t], thdot[t]} /. nd[[1]], {t, 0, tmax}];
```

In[3149]:=
```
tmax = 10;
th0 = -91 * Pi / 180;
thS = -80 * Pi / 180;
nd = NDSolve[{theta'[t] == thdot[t], 0 == (D[pp, t] - ff) /. theta0 → th0,
    thdot[0] == 0, theta[0] == thS}, {theta[t], thdot[t]}, {t, 0, tmax}];
plot80 = ParametricPlot[{theta[t], thdot[t]} /. nd[[1]], {t, 0, tmax}];
```

In[3159]:=
```
tmax = 10;
th0 = -91 * Pi / 180;
thS = -70 * Pi / 180;
nd = NDSolve[{theta'[t] == thdot[t], 0 == (D[pp, t] - ff) /. theta0 → th0,
    thdot[0] == 0, theta[0] == thS}, {theta[t], thdot[t]}, {t, 0, tmax}];
plot70 = ParametricPlot[{theta[t], thdot[t]} /. nd[[1]], {t, 0, tmax}];
```

In[3165]:=
```
Show[plot90, plot80, plot70]
```

Out[3165]=