Physics 422 – Fall 2012

Homework #10, due at beginning of class **Friday Nov 30**.

1. [8 pts] Three equal masses have total kinetic energy

$$T = \frac{1}{2}M\left(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2\right) .$$

The potential energy is

$$V = \frac{1}{2}\left[K_a(x_1^2 + x_3^2) + K_b\, x_2^2 + K_c\,(x_1 + x_3)x_2\right] .$$

Find the normal modes if $K_c = \sqrt{2K_aK_b}$.

2. [8 pts] A thin hoop of radius R and mass M hangs from a single fixed point and oscillates in its own plane. A mass m slides without friction along the hoop (near the bottom). Find the two eigenfrequencies for small oscillations, and find initial conditions that will cause the system to oscillate in each of these normal modes.

A convenient choice of coordinates would be θ_1 = the angle with respect to vertical of the line connecting the pivot point to the center of the hoop, and θ_2 = the angle with respect to vertical of the line connecting the center of the hoop to the sliding mass.

This problem is similar to Marion & Thornton problem 12.16, except that M&T assume $M = m$. You can use the answer given in that problem to check your answer in the $M \to m$ limit.

(Last updated 11/19/2012.)