Physics 422 – Fall 2012

Homework #8, due at beginning of class Friday Nov 9.

1. [5 pts] Review problem: in a particular coordinate frame, the moment of inertia tensor of a rigid body is given by

$$\mathbf{I} = \begin{pmatrix} 3 & 4 & 0 \\ 4 & 9 & 0 \\ 0 & 0 & 12 \end{pmatrix}$$

in some units. The instantaneous angular velocity vector in that frame is given by $\omega = (2, 3, 4)$ in some units.

   (a) Find the principal moments of inertia.

   (b) Find a rotation matrix a that transforms to a new coordinate system in which the moment of inertia tensor becomes diagonal.

   (c) Find the moment of inertia tensor $\mathbf{I}'$ and the angular velocity vector ω' in the new coordinate system.

   (d) Compute the kinetic energy and the magnitude of the angular momentum in both frames, and compare the results.

2. [5 pts] Three springs are connected end-to-end, with point masses M_1 and M_2 at the junctions between springs. The outer ends of the springs are connected to rigid walls. The only allowed motion is along the line of the springs. Let x_1 and x_2 be the displacements of M_1 and M_2 from their equilibrium positions. This problem is the same as a problem worked out in lecture, except that the masses are now different.

   Assume that the masses are $M_1 = 5$ and $M_2 = 12$ in some units. Assume that the spring constants are $K_0 = 1$ for the two outer springs, and $K = 1/3$ for the middle spring in some units.

   (a) Find the normal mode frequencies of oscillation.

   (b) Find the most general motion ($x_1(t)$ and $x_2(t)$) for the system.

   (c) Find the motion of the system for the initial conditions

$$\begin{aligned} x_1(0) &= 1 \\ x_2(0) &= 0 \\ \dot{x}_1(0) &= 0 \\ \dot{x}_2(0) &= 0 \end{aligned}$$

3. [5 pts] A particle of mass M is free to move in 3-dimensional space, subject to the potential

$$V(x, y, z) = (4x^2 + 5y^2 + 6z^2 - 2ax - 5ay + 8xy)\, V_0/a^2$$

where V_0 and a are positive constants. Find the normal mode frequencies for oscillations about the minimum of this potential.

   (Hint: Begin by finding the location of the minimum of the potential, and introducing new coordinates that are displacements from that point.)

(Last updated 11/05/2012.)