Physics 422 – Fall 2012

Homework #9, due at beginning of class Friday Nov 16.

Reminder: Exam #3 on Monday Nov 19; no class Wednesday Nov 21.

1. [8 pts] A triple pendulum is made from three equal point masses ($M = 1$). The first mass hangs by a massless string attached to a fixed point; the second mass hangs by a massless string attached to the first mass; and the third mass hangs by a massless string attached to the second mass. All three strings have the same length $\ell = 1$.

   Use the angles θ_1, θ_2, θ_3 of each string with respect to the vertical direction as generalized coordinates.

   (a) Find the normal mode frequencies of this system for small oscillations.

   (b) Find the relative displacements in (θ_1, θ_2, θ_3) for each of the three normal modes. (These are the quantities that were called $Q_i^{(n)}$ in lecture.)

   You may use an electronic aid such as Mathematica to handle the details of this. But if you choose that option, make sure what you hand in describes what you did explicitly; or better yet, include the input and output from Mathematica, or whatever program you use.

2. [8 pts] Five identical springs (each with spring constant $K = 1$) are connected end-to-end, with identical point masses $M = 1$ at the four junctions, and the outer ends are attached to rigid walls.

   (a) Find the normal mode frequencies of oscillation.

   (b) Find the most general motion ($x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$) for the system.

   (c) Find the motion of the system for the initial conditions

$$\begin{aligned} x_1(0) &= 1 \\ x_2(0) &= 0 \\ x_3(0) &= 0 \\ x_4(0) &= 0 \\ \dot{x}_1(0) &= 0 \\ \dot{x}_2(0) &= 0 \\ \dot{x}_3(0) &= 0 \\ \dot{x}_4(0) &= 0 \end{aligned}$$

(Last updated 11/12/2012.)