PDF Parametrization issues

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Outline

- Parametrization dependence
- Example: strangeness in CTEQ6.6
- How to include theory constraints in PDF fit
- Remarks on Regge behavior at small x

PDF fitting paradigm

1. Parameterize each flavor $f_a(x, Q)$ at fixed small Q_0 $(A_1, \ldots, A_{22})$
2. Compute PDFs $f_a(x, Q)$ at $Q > Q_0$ by DGLAP
3. Compute cross sections for DIS(e,μ,ν), Drell-Yan, Inclusive Jets,...
4. Compute "χ^2" measure of agreement between predictions and measurements:

$$\chi^2 = \sum_i W_i \left(\frac{\text{data}_i - \text{theory}_i}{\text{error}_i} \right)^2$$

   generalized to include correlated systematic errors.
5. Minimize χ^2 with respect to the shape parameters $\{A_i\}$
6. Uncertainty Range is the region in $\{A_i\}$ space where χ^2 is sufficiently close to minimum that all experiments are fit tolerably well—characterized by eigenvector sets.

Parametrization dependence = systematic error from the functional choices in $f_a(x, Q_0)$.

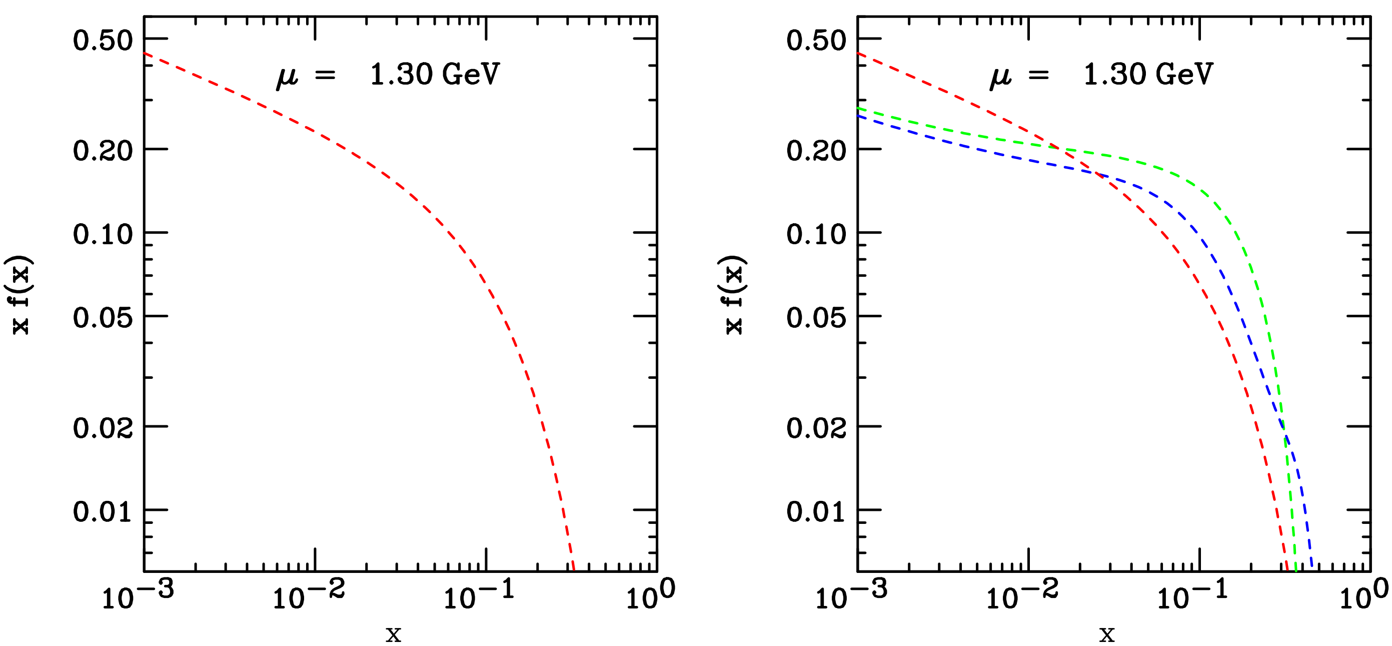
(Alternative: could just define $f_a(x, Q_0)$ on a few grid points and interpolate — neural net method.)

Example from CTEQ6.6

Previous CTEQ PDF analysis generally assume $s(x) = \bar{s}(x) \propto \bar{d}(x) + \bar{u}(x)$ at Q_0. We dropped that Ansatz in CTEQ6.6.

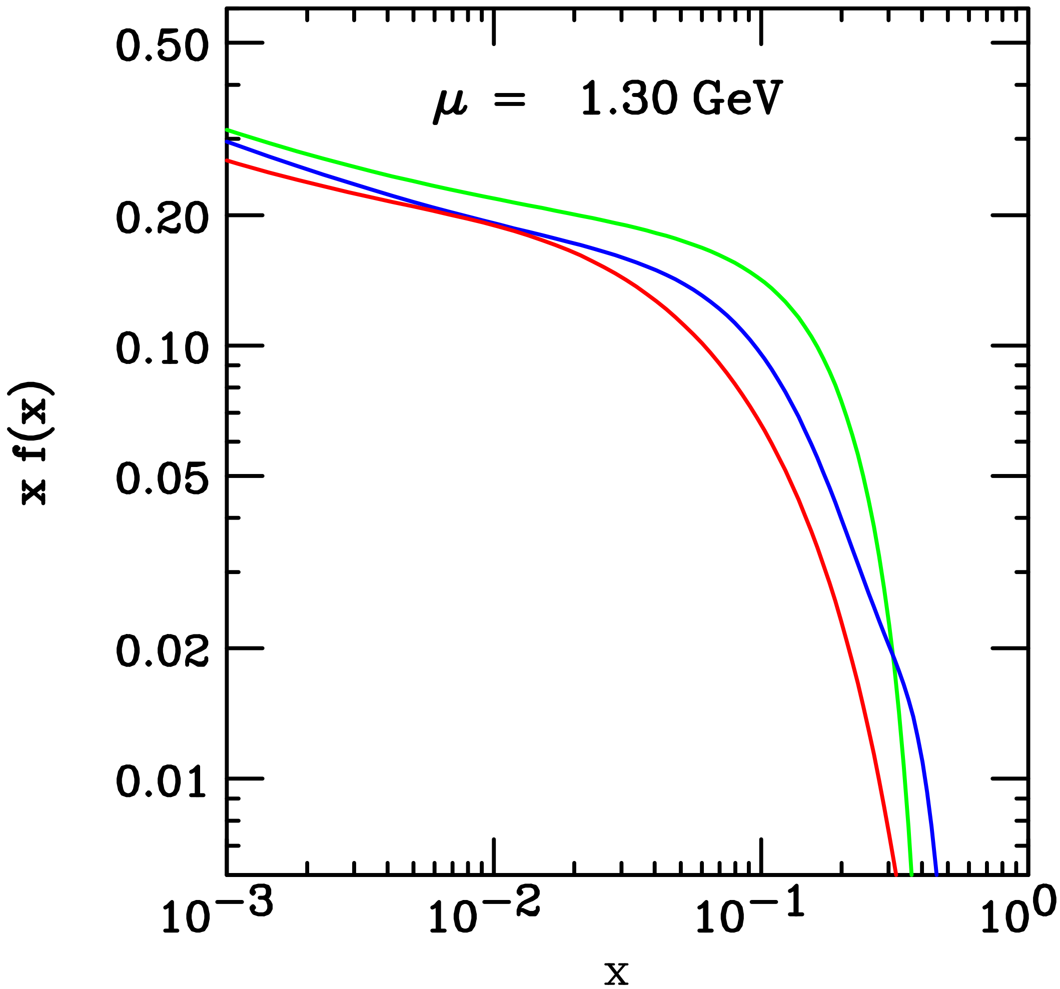
A preliminary version used innocent-looking form $s(x) = \bar{s}(x) = a_0\, x^{a_1}\, (1 - x)^{a_2}$, with a_1 same as for $\bar{d}$ and $\bar{u}$ as given by Regge theory.

Strangeness looked OK by itself; but $\bar{s}(x) > \bar{u}(x)$, $\bar{d}(x)$ at small x—violates theory prejudice and perhaps Hermes data.



Red $= \bar{s}$, Blue $= \bar{u}$, Green $= \bar{d}$.

More elaborate parametrization used in CTEQ6.6 chosen to make $\bar{s}(x, Q_0)$ smaller at small x:



Red = $\bar{s}$, Blue = $\bar{u}$, Green = $\bar{d}$.

General method for theory constraints

Example: use a very flexible parametrization such as $\bar{s}(x) = a_0\, x^{a_1}\, (1-x)^{a_2}\, e^{a_3\sqrt{x} + a_4 x}$ that has more parameters than can be determined from the data.

Then add a "penalty" to χ^2 to force parameters such as a_2 and $\bar{s}(x)/(\bar{d}(x)+\bar{u}(x))$ at $x \to 0$ to fall within the range allowed by our theoretical prejudices.

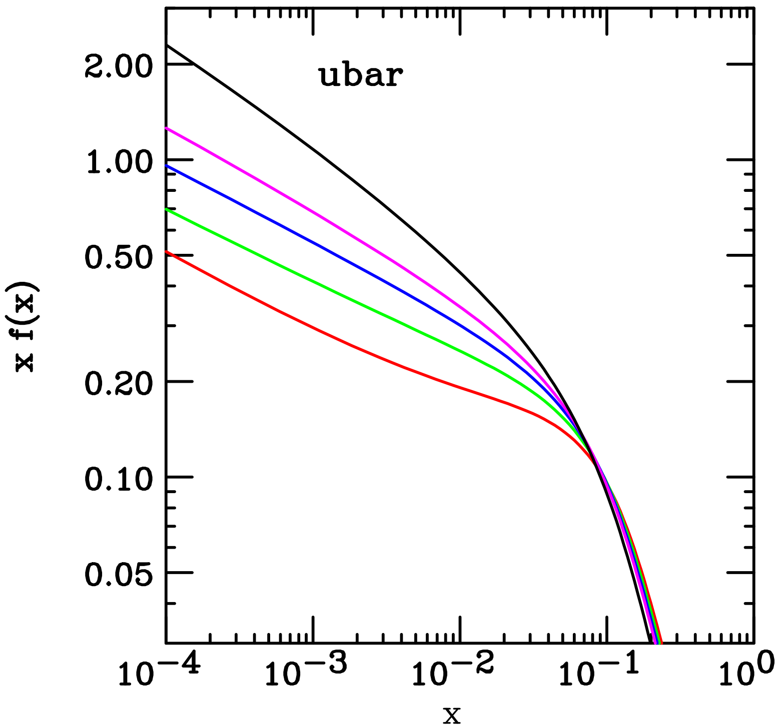
For the central "Best Fit", this is no different from the previous method of just freezing parameters that cannot be determined from the fitting at plausible values. But for the uncertainty analysis, it captures the actual wider range of uncertainties, since a new eigenvector direction (two new extreme eigenvector sets) is generated for each parameter that is treated this way.

In the standard analysis, there are several parameters, such as the $(1-x)^{a_2}$ behavior of $g(x, Q_0)$ or $d_v(x, Q_0)$ which could benefit from this treatment.

The goal in all this is to avoid repeating the "HJ" scenario, in which new data (Run I inclusive jet data at Tevatron) appeared to lie outside of standard-model predictions; but were later found consistent using an unanticipated form for $g(x, Q_0)$.

Regge behavior of $\bar{u}(x)$

The Regge behavior $f(x,Q) \propto x^{a_1}$ that we assume for $x \to 0$ at Q_0 is quite well preserved by DGLAP evolution. This can be seen by the nearly straight-line behavior on a log-log plot, with slope nearly independent of Q:



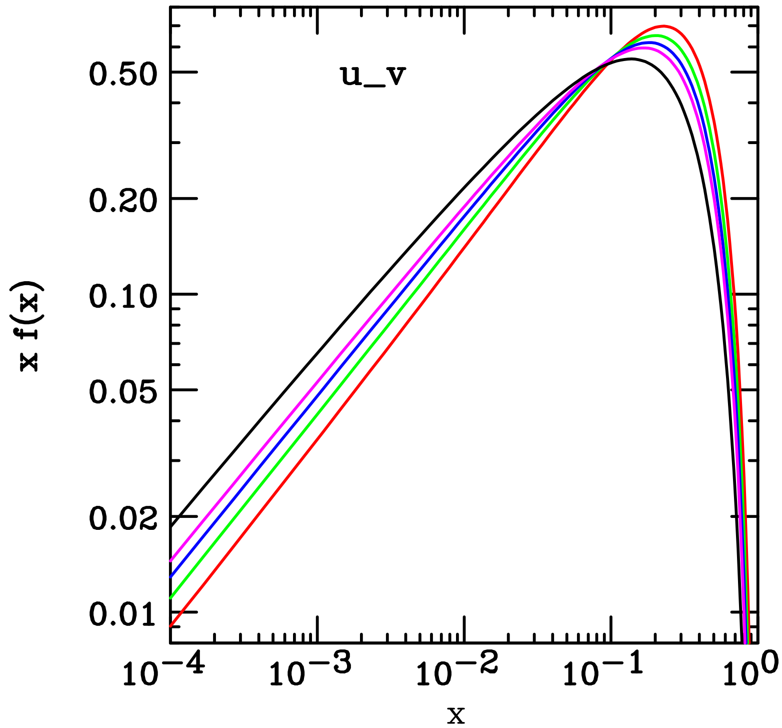
Where Red/Green/Blue/Magenta/Black:
$Q = 1.3/2.0/3.2/5.0/20$ GeV.

The numerical value of the slope a_1 agrees well with expectations from Regge. That result supports the use of the $f(x,Q) \propto x^{a_1}$ ansatz.

However, the uncertainty in a_1 from fitting is small compared to the uncertainty of estimates based on Regge, so the theory does not provide a useful constraint.

Regge behavior of $u_v(x)$

The Regge behavior $f(x,Q) \propto x^{a_1}$ that we assume for $x \to 0$ at Q_0 is also well preserved by DGLAP evolution:



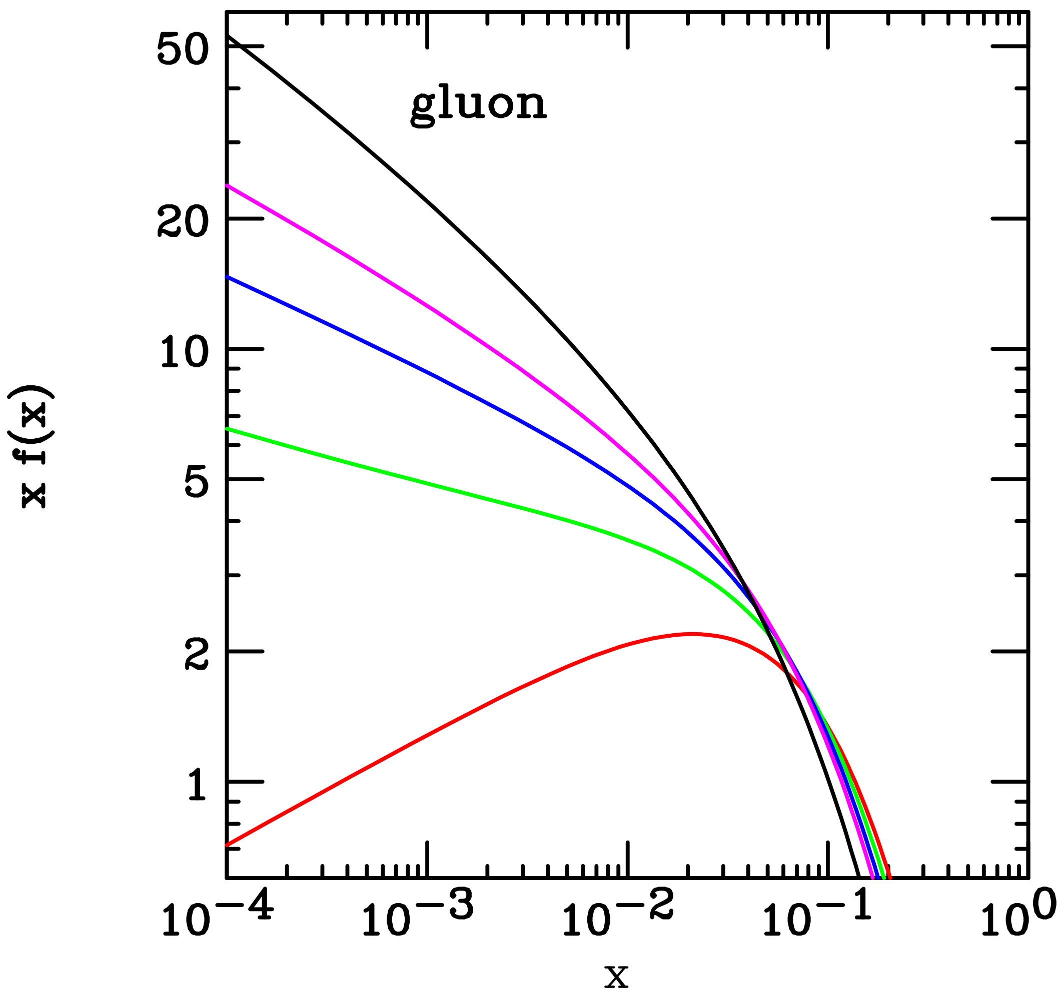
where Red/Green/Blue/Magenta/Black: $Q = 1.3/2.0/3.2/5.0/20\,\text{GeV}$.

Again the observed slope value a_1 is consistent with expectations from Regge theory, which supports the choice of functional form.

However, again the uncertainty in a_1 from PDF fitting is small compared to the uncertainty of its estimate based on Regge theory, so traditional Regge phenomenolo doesn't provide a useful constraint on a_1 to improve PDFs.

$g(x)$ at small x

In contrast to valence and sea quark distributions, the NLO evolution of the gluon distribution at small x is very rapid, so no simple comparison can be made with expectations from Regge theory:



where Red/Green/Blue/Magenta/Black: $Q = 1.3/2.0/3.2/5.0/20$ GeV.

Perhaps small-x resummation corrections to DGLAP would restore Regge behavior for $g(x, Q)$?