Dmitri Ryvkine - Statement of Research Interests

November 22, 2005

My current research interests lie in the areas of (i) activated processes in nonequilibrium systems and (ii) microwave absorption and heating of 2D electrons on liquid helium. The work I have been doing with my advisor Mark Dykman is theoretical and involves both analytical and numerical methods, as well as simulations.

Activated escape underlies many physical, biological and, arguably, social phenomena. It leads to switching between coexisting metastable states. Since interstate switching requires comparatively large fluctuations, it becomes increasingly important with reduction of the system size, as in micro- and nano-scale systems. While it is generally understood how escape happens in equilibrium, it is still largely an open question for systems away from equilibrium, such as those modulated by time-dependent fields. In the course of our work we established previously unknown universal features of activated escape in nonequilibrium systems. In particular we investigated scaling of the activation energy and the pre-exponential factor of the escape rate near a bifurcation point, time dependence of the escape current, and dynamics of escape pathways in periodically modulated systems. Below I highlight the accomplished and ongoing projects, and possible future directions of this research.

1. Scaling of the activation energy near a bifurcation point

In equilibrium systems, the rate of noise-induced activated escape from a metastable state is $W \propto \exp(-R/D)$, where R is the activation energy, and D is the noise intensity. For thermal fluctuations $D = k_BT$ and the activation energy R is given by the height of the free energy barrier. When the system approaches a saddle-node bifurcation point (spinodal point) where the energy barrier disappears, the activation energy scales as $R \propto |A - A_c|^{\xi}$, with $\xi = 3/2$. Here A is the control parameter, e.g. the strength of the applied DC field, and A_c is the critical value of A. This scaling is related to the universality of critical dynamics near the bifurcation point, where the effective potential becomes locally cubic. In periodically modulated systems, the period-averaged escape rate has a similar form, $\overline{W} \propto \exp(-R/D)$, but the activation energy R is not equal to the barrier height. The most interesting case is that of sufficiently slow modulation, where dynamics away from the critical point can be well described in the adiabatic approximation. Due to the critical slowing down, the adiabatic approximation necessarily breaks down close to $A = A_c$. As we have shown, this breakdown of adiabaticity occurs in a universal way. Sufficiently far from A_c the adiabatic scaling with $\xi = 3/2$ is observed. For smaller $|A_c - A|$ there emerges a new dynamical time scale and a new, locally nonadiabatic scaling of the activation energy with $\xi = 2$. For even smaller $|A_c - A|$ the exponent goes back to $\xi = 3/2$. Thus, scaling of the activation energy in periodically modulated systems involves crossovers between three scaling exponents $\xi = 3/2, 2, 3/2$ instead of a single exponent $\xi = 3/2$ in the equilibrium case. The widths of the three regions with different scaling depend on the modulation frequency. The new dynamical scaling with $\xi = 2$ is a consequence of nonadiabatic retardation effects and has no analogue in equilibrium systems.

2. Time dependence of the escape current

In equilibrium systems the escape rate is independent of time and has the form $W = \nu \exp(-R/D)$, where the prefactor ν is given by the generalized "attempt frequency" in the metastable potential well. In periodically modulated systems $W(t)$ is a periodic function of time. It is given by the current $j(q,t)$ away from the metastable state, which also depends on the position q where it is measured. It is physically meaningful to measure the current sufficiently far behind the boundary of the basin of attraction to the metastable state. We calculate the escape current using the eikonal approximation and methods of nonlinear Hamiltonian dynamics and matched expansions. We identify a single parameter θ that determines the shape of the periodic pulses of the escape current as functions of time. It depends on both the strength and the frequency of the modulation. In the limit of adiabatically slow modulation or small $|A - A_c|$ we have $\theta \ll 1$, and the current pulses have Gaussian shape with the width much smaller than the modulation period. For $\theta \gg 1$ the pulses are strongly non-Gaussian and asymmetric.

3. Scaling of the pre-exponential factor of the period-averaged escape rate

We have explored the prefactor ν of the period-averaged escape rate close to the bifurcation point $A = A_c$ where the periodic metastable state disappears. We have found that the prefactor displays three regions of system-independent scaling corresponding to the scaling regions of the activation energy, with $\nu \propto |A - A_c|^{\zeta}$. The scaling exponents are $\zeta = 1/4, -1, 1/2$. The exponent $\zeta = 1/4$ can be observed for sufficiently slow modulation not too close to the bifurcation point. The negative exponent $\zeta = -1$ is a consequence of the onset of local nonadiabaticity, which leads to the exponent $\xi = 2$ for the activation energy. The exponent $\zeta = 1/2$ can be observed for fast modulation or very close to the bifurcation point.

4. Dynamics of pathways of activated escape

One of important questions regarding fluctuating systems is how a fluctuation evolves in time. For instance, if we find a system at a phase-space point far away from equilibrium it might be of interest to know how the system was moving on the way to this point. This knowledge gives one the power to control fluctuations in the system, e.g. to increase or suppress the probability to visit certain areas of the phase space by applying external fields. Control of infrequent events such as escape from a metastable state is facilitated by the fact that, in fluctuations leading to such events the system is most likely to follow a certain dynamical path. Fluctuational trajectories form a narrow tube centered at this path. We have developed a formalism for studying these tubes in periodically modulated systems. We found, and confirmed by detailed simulations that, in contrast to stationary systems, periodically modulated systems exhibit several tubes of trajectories leading to escape at a given time. The tubes have specific shapes and intensities, which we have described analytically. The spatial and temporal localization of the trajectories suggests the ways of optimal and selective control of escape events.

5. Hysteresis of microwave absorption by 2D electrons on liquid helium

A two-dimensional electron system on helium surface provides an example of a strongly correlated Coulomb liquid and has attracted significant interest of theorists and experimentalists. Recent experiments have revealed the possibility to observe saturation of intersubband absorption in this system. Our theoretical results show that absorption saturation may be accompanied by hysteresis. The hysteretic behavior sensitively depends on the electron-electron interaction and the relaxation mechanisms. Studying it should provide a new important insight into the physics of the system, which is of interest for various applications, including quantum computing.

6. Possible future directions

One of immediate applications of the theory of activated processes in the presence of time-dependent fields is the problem of control. By applying external fields one can exponentially suppress or enhance the probability of switching events in systems with coexisting metastable states. The fact that escape occurs along well-defined pathways suggests where and when the control field must be applied and makes the control highly selective. Besides control, a natural direction of further research is an extension of the ideas and techniques developed for single-particle systems to systems with many particles, multi-dimensional systems, and systems where fluctuations are induced by shot noise. Another challenging extension is theory of activated processes in quantum systems far away from thermal equilibrium, including systems used in quantum information and quantum computation. Also, activated processes are fundamentally important in biosystems; selective control of noise-induced switching in biosystems is another broad area of possible future effort.