Interjet Energy and Color Flow

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Rapidity gap events are conventionally defined by requiring no particle production in a gap region. In the case of dijet events, the distribution of energy, $E_{\rm gap}$, that flows into the interjet region is calculable in perturbative QCD nearly down to $E_{\rm gap} = 0$, and sheds light on the role of color exchange in hard scattering. Distributions are calculable for $E_{\rm gap}$ as a function of scattering angle, momentum transfer and gap width. The concept of a hard color singlet exchange is clarified.

1. GAPS AND COLOR FLOW

A compelling heuristic principle suggests that the exchange of gluons in a color singlet state produces little radiation even when the scattering produces jets at high p_T [1]. The correspondence has such currency that dijet gap events are routinely termed "color singlet exchange". This insight, however, has been difficult to implement in perturbative terms. After all, gluons of any energy carry octet color charge, so that there is no unique way of defining color exchange in a finite amount of time [2]. On the other hand, it takes a very short time to radiate a hard gluon, and once radiated, it cannot be reabsorbed on the basis of soft color rearrangements at very long times.

1.1. Two-gluon vs. Soft Color Models

The simplest short-distance model for dijet gaps is based on two-gluon exchange [1]. In a two-gluon model, the gap is usually filled by spectator interactions, up to a "survival probability", P_S, which may be estimated [1,3] from low-p_T diffractive scattering to be of order one tenth. Denoting the probability for hard color-singlet exchange as f_1, the fraction of gap events becomes

$$f_{\rm gap} = f_1 \, P_S \, . \tag{1}$$

If we estimate $f_1 \sim \mathcal{O}(\alpha_s(p_T)/\pi) \sim 0.1$, we predict gap events at the one percent level, and this is what is seen experimentally [4–7]. This analysis would lead us to expect more gap events from gluon-gluon than quark-quark scattering, because of the larger color factors in exchange graphs between gluons. This expectation was tested by comparing 630 and 1800 GeV data from the Tevatron, because at fixed p_T the role of gluon-gluon scattering increases with the overall center-of-mass energy. The proportion of gap events, however, decreased, rather than increased, with the energy.

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In the soft color approach [8], normally presented as an alternative to the two-gluon model, the underlying hard scattering is treated at lowest order, which for gap events is primarily single-gluon, color octet exchange. The gap probability is determined by counting possible color exchanges, assuming all to be equally likely, up to an overall survival factor (rather larger than 1/10). Because gluons have more color states than quarks, they are correspondingly less likely to produce gap events. The soft color model then naturally leads to fewer gap events as the energy, and hence the role of gluons, increases.

1.2. Energy Flow

A third approach is a perturbative QCD formalism for rapidity gaps, made possible by redefining gaps in terms of an energy flow, $E_{\rm gap}$, rather than particle multiplicty [9]. The resulting cross sections can be treated via standard factorization theorems. In this formulation, if $E_{\rm gap} \gg \Lambda_{\rm QCD}$ the cross section is perturbatively calculable. In addition, when $p_T \gg E_{\rm gap} \gg \Lambda_{\rm QCD}$, the gap cross sections have two perturbative scales, and logarithms in their ratio can be resummed by renormalization group methods.

Resummation in $\ln(E_{\rm gap}/p_T)$ allows us to probe color flow at short distances, and to generalize the concept of hard color singlet exchange.

The dijet cross section at measured $E_{\rm gap} \gg \Lambda_{\rm QCD}$ falls into the class of inclusive jet cross sections that can be written in factorized form:

$$\frac{d\sigma}{dE_{\rm gap}\, d\cos\hat\theta}\,(\Delta y) = \sum_{f_A,f_B} \phi_{f_A/p} \otimes \phi_{f_B/\bar p} \otimes \sum_{f_C,f_D} \frac{d\hat\sigma^{({\rm f})}}{dE_{\rm gap}\, d\cos\hat\theta}\,(\Delta y)\,, \tag{2}$$

with the $\phi_{f/h}$ parton distributions, evaluated at the scale of the dijet momentum transfer. The partonic cross section $d\hat\sigma^{({\rm f})}/dE_{\rm gap}\, d\cos\hat\theta$ is a hard scattering function, starting with the Born cross section at lowest order (cf. the soft color model). The index f denotes the partonic hard scattering $f_A + f_B \to f_C + f_D$. The cross section depends on the dijet pair rapidity y_{JJ}, the partonic center-of-mass (c.m.) energy squared $\hat{s}$, the partonic c.m. scattering angle $\hat{\theta}$, with $-\frac{\hat{s}}{2}\left(1-\cos\hat{\theta}\right) = \hat{t}$, and Δy, the gap size as a rapidity interval.

2. INTERJET ENERGY DISTRIBUTION

To leading logarithm in $E_{\rm gap}/\sqrt{-\hat{t}}$, the gap energy dependence is of the form

$$\frac{d\hat{\sigma}^{(\rm f)}}{dE_{\rm gap}\, d\cos\hat{\theta}}\,(\Delta y) \;=\; \sum_{\beta\gamma} \tilde{\sigma}_{\beta\gamma}(\hat{\theta}) \times \frac{E_{\gamma\beta}}{E_{\rm gap}} \left[\ln\left(\frac{E_{\rm gap}}{\Lambda}\right)\right]^{E_{\gamma\beta}-1} \left[\ln\left(\frac{\sqrt{-\hat{t}}}{\Lambda}\right)\right]^{-E_{\gamma\beta}} . \quad (3)$$

In this expression, β and γ label the color exchange at short distances, contained in $\bar{\sigma}$, in a color tensor basis that diagonalizes a perturbative anomalous dimension matrix. The exponents $E_{\gamma\beta}$ are given in terms of the eigenvalues, $\lambda_\beta = (\alpha_s/\pi)\lambda^{(1)}_\beta$, of this matrix by

$$E_{\gamma\beta}\left(\hat{\theta},\Delta y\right) = \frac{2}{\beta_0}\left[\lambda^{(1)*}{}_{\gamma}, + \hat{\lambda}^{(1)}_{\beta}\right] , \quad (4)$$

with $\beta_0 = 11N_c/3 - 2n_f/3$. The color exchange with smallest eigenvalue thus dominates the behavior of the cross section in the limit $E_{\rm gap}/p_T \to 0$.

The concept of a dominant eigenvalue generalizes conventional hard singlet exchange, because the eigenvectors of the anomalous dimension matrix are linear combinations of elements in the basis of t-channel color transfers. The coefficients depend, in general, on the jet scattering angle. Eq. (3) thus leads to a detailed set of predictions for dijet data with measured interjet energy flow.

Explicit anomalous dimension eigenvalues λ_α for quark and gluon processes may be found in Ref. [10]. The overlap of the dominant eigenvector with hard color singlet exchange grows in the direction of forward scattering for all partonic processes, so that in the Regge limit, $-\hat{t}/\hat{s} \to 0$, $\hat{t}$ fixed, the dominant color exchange becomes purely color singlet [11]. In addition, the eigenvalues for gluon-gluon scattering are larger than those for processes involving quarks. This makes it harder for gluon-gluon hard scattering to produce rapidity gaps, for much the same reasons as in the soft color model: the size of the eigenvalue λ_α is related to the number of color states available. At low interjet energy, however, the smallest eigenvalue for quark-antiquark scattering is actually less than unity in absolute value, and produces a small upturn in the interjet energy distribution in predictions based on Eq. (3) [9]. This is the "hard singlet exchange" observed by CDF and D0 [4,5]. Gap events defined by vanishing particle multiplicity in the interjet region are counted in this excess. The perturbative predictions for such events must be diluted, as usual, by corrections associated with spectator interactions, which, according to the factorization formalism, are suppressed only by powers of $\Lambda_{\rm QCD}/E_{\rm gap}$, and which therefore become important for small $E_{\rm gap}$. We have in Eqs. (2)-(4), however, a set of predictions for the full range of $E_{\rm gap}$.

3. SUMMARY

Energy flow analysis makes possible a quantitiative study of radiation in interjet regions, and gives a perturbative meaning to short-distance color exchange, generalizing both the two-gluon and soft color models. On the basis of this analysis, gaps in dijet events come from a compound structure, predominantly, but not purely, singlet in the hard scattering [9,10]. Many qualitative results, including the relative suppression of dijet gaps for gluon-gluon scattering, are consistent with the successes of the soft color model. The perturbative analysis offers a systematic set of differential predictions for energy flow, as a function of momentum transfer, flavor and interjet rapidity interval.

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