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$H \to ggg(gq\bar{q})$ at Two Loops in the Large-M_t Limit

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Abstract

We present a calculation of the two-loop helicity amplitudes for the processes $H \to ggg$ and $H \to gq\bar{q}$ in the large-M_t limit. In this limit the calculation can be performed in terms of one-loop diagrams containing an effective Hgg operator. These amplitudes are required for the next-to-leading order (NLO) corrections to the Higgs transverse momentum distribution and the next-to-next-to-leading order (NNLO) corrections to the Higgs production cross section via the gluon fusion mechanism.

1 Introduction

The Higgs Boson H is the only particle of the Standard Model remaining to be discovered. Its role is to provide a simple mechanism to break the electroweak gauge symmetry and to give mass to the weak gauge bosons and the fermions. Of course, it is possible that nature uses more than a single scalar boson for this purpose, but still the Standard Model and its supersymmetric extension are the primary examples of the class of symmetry-breaking models which interact weakly. Therefore, the search for the Higgs boson is of the highest priority for the Large Hadron Collider (LHC) at CERN.

The detection of the Higgs Boson above background at the LHC will be a challenging task. In particular, if the mass of the Higgs is below ~ 140 GeV, near the threshold for decay into W boson pairs, the detection of the Higgs is quite subtle. Although the largest production mechanism by far is gluon-gluon fusion, the equally large backgrounds require the use of the $H \to \gamma\gamma$ decay channel, which has a branching ratio of $\mathcal{O}(10^{-3})$. To prepare for the search, we need the best theoretical predictions possible, and this means the inclusion of quantum chromodynamic (QCD) corrections to Higgs production and decay. Recent relevant reviews are given in reference [1].

The Higgs production via gluon-gluon fusion proceeds at lowest order (LO) through a quark loop. This loop is dominated by the top quark, because the Higgs coupling is proportional to the quark mass. The two-loop next-to-leading order (NLO) QCD corrections have also been calculated [2], and they are quite large:

$\sim 50-100\%$. An interesting feature of this NLO calculation is that it becomes much simpler in the limit of large top quark mass ($M_t \to \infty$). In this limit, one can integrate out the heavy top quark loop, leaving behind an effective gauge-invariant Hgg coupling. Thus, the number of loops at each order is reduced by one. It has been shown that the NLO corrections in this large-M_t limit give a good approximation to the complete two-loop result over a large range of Higgs masses [3]. The large NLO correction suggests that even higher orders still may be important. A soft-gluon resummation in the large-M_t limit has recently been performed, which gives an estimate of the next-to-next-to-leading order (NNLO) corrections [4].

Meanwhile, other groups have considered less inclusive quantities, such as the transverse momentum spectrum of the Higgs boson. This observable has been considered at the one-loop Born level, both in the large-top-mass limit and with full M_t dependence [5]. In addition, the effects of soft gluons have also been studied, which modify the spectrum at small Higgs $p_\perp$ [6]. However, a NLO calculation has not been done. The real five-point $H \to gggg$ amplitudes which are needed have been calculated by Dawson and Kauffman [7] in the large-M_t limit, and recently Kauffman et al. [8] have calculated the five-point amplitudes with light external quarks.

In this paper we present the virtual corrections to the four-point $H \to ggg(gq\bar{q})$ amplitudes in the large-M_t limit, which completes the set of amplitudes needed to study the Higgs $p_\perp$ spectrum at NLO. In this limit, the two-loop results can be computed from effective one-loop diagrams. The large-M_t approximation to the Higgs $p_\perp$ spectrum will be good for some range of M_H and Higgs $p_\perp$, and furthermore this calculation offers a check of the complete M_t-dependent result, when it should become available. Moreover, these amplitudes are necessary for a full NNLO calculation of the cross section in the large-M_t limit [9].

In addition to using the effective Higgs-gluon operator in the large-M_t limit, we have also used several other techniques that have been found convenient in QCD loop calculations [10]. These include the use of helicity spinors, color ordering, and background-field gauge. In section 2 we discuss the details of the calculation, while in section 3 we present the amplitudes and discuss various cross-checks. In section 4 we summarize our conclusions.

2 Calculational Details

In the large-M_t limit the top quark can be removed from the full theory, leaving a residual Higgs-gluon coupling term in the lagrangian of the effective theory:

$$\mathcal{L}_{\text{eff}} \;=\; -\frac{1}{4}\Big[1-\frac{\alpha_s}{3\pi}\frac{H}{v}(1+\Delta)\Big]\,\mathrm{Tr}\,G_{\mu\nu}G^{\mu\nu}\;. \qquad (1)$$

The finite $\mathcal{O}(\alpha_s)$ correction to this effective operator has been calculated [3] to be

$$\Delta \;=\; \frac{11\alpha_s}{4\pi}\;. \qquad (2)$$

Following Mangano and Parke [11], we use the unconventional normalization for the $SU(3)$ generator matrices $\mathrm{Tr}(T^aT^b)=\delta^{ab}$ and $[T^a,T^b]=i\sqrt{2}f^{abc}T^c$. This will remove factors of $\sqrt{2}$ from the helicity amplitudes below.

As suggested by string theory methods [10], we use the background-field gauge to calculate the one-particle irreducible parts of the Feynman diagrams. The gluon field G^μ is split into a background component B^μ and a quantum component Q^μ, *i.e,* $G^\mu = B^\mu + Q^\mu$. Two reasonable choices for the gauge-fixing term are

$$\begin{aligned} \mathcal{L}^{(1)}_{\rm gf} &= -\frac{1}{2}(D^B_\mu Q^\mu)^2 \\ \mathcal{L}^{(2)}_{\rm gf} &= -\frac{1}{2}\Big[1 - \frac{\alpha_s}{3\pi}\frac{H}{v}(1+\Delta)\Big](D^B_\mu Q^\mu)^2 \end{aligned} \tag{3}$$

where $D^B_\mu Q^\mu = \partial_\mu Q^\mu - (ig/\sqrt{2})[B_\mu\,,\, Q^\mu]$ is the background-field covariant derivative of the quantum field. The second choice of gauge-fixing condition has the advantage that the $Hggg$ and the $Hgggg$ Feynman vertices retain the same structure as the ggg and $gggg$ vertices. The simple organization of these vertices is the feature that makes the background-field gauge the preferred gauge for doing one-loop calculations in QCD [10]. However, the price to be paid is that the Higgs boson now couples to the ghost fields. We have verified that our results are the same using either gauge-fixing term.

We first consider the $H \to ggg$ amplitude. For simplicity we take all particles to be outgoing so that we actually calculate the amplitude for the process $0 \to Hg_1g_2g_3$ with $p_H + p_1 + p_2 + p_3 = 0$. The amplitude for gluons with helicities λ_i and colors a_i can be written

$$\mathcal{M} = -\frac{\alpha_s}{3\pi v}\frac{g_s}{2}\,{\rm tr}(T^{a_1}[T^{a_2}, T^{a_3}])\, m(p_1,\lambda_1;p_2,\lambda_2;p_3,\lambda_3)\ . \tag{4}$$

Note that the color ordering of the amplitudes is trivial here.

It is convenient to write these helicity amplitudes in terms of products of Weyl spinors $|p\pm\rangle$. The polarization vector for an outgoing gluon of momentum p can be written [12]

$$\epsilon_\pm(p)^\mu \;=\; \frac{\pm\langle p\pm|\gamma^\mu|q\pm\rangle}{\sqrt{2}\langle q\mp|p\pm\rangle}\;. \tag{5}$$

The arbitrary reference vector q satisfies $q^2 = 0$ and $q \cdot p \neq 0$. A change in the reference vector just shifts $\epsilon(p)^\mu$ by a term proportional to p^μ, which drops out of the gauge-invariant helicity amplitude. There are two independent $Hggg$ helicity subamplitudes, which at tree level are

$$\begin{aligned} m^0(1^+,2^+,3^+) &= \frac{-M_H^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle} \\ m^0(1^-,2^+,3^+) &= \frac{[2\,3]^4}{[1\,2][2\,3][3\,1]}\;. \end{aligned} \tag{6}$$

Here we have used the notation $\langle ij\rangle = \langle p_i - |p_j+\rangle$ and $[ij] = \langle p_i + |p_j-\rangle$. These spinor products are antisymmetric and satisfy $\langle ij\rangle[ji] = 2p_i \cdot p_j \equiv S_{ij}$. All other subamplitudes can be obtained by invariance under cyclic permutations, charge conjugation, and parity.

We also consider the process $0 \to Hgq\bar{q}$ with $p_H + p + q + \bar{q} = 0$. The amplitude for a gluon and quarks with helicities $\lambda, h, \bar{h}$ and colors $a, i, \bar{\imath}$ can be written

$$\mathcal{M} \;=\; -\frac{\alpha_s}{3\pi v}\,\frac{g_s}{2}\,T^a_{i\bar{\imath}}\,m(p,\lambda;q,h;\bar{q},\bar{h})\;. \tag{7}$$

At tree level we have

$$m^0(g^+,q^-,\bar{q}^+) \;=\; \frac{[p\bar{q}]^2}{[\bar{q}q]}\;. \tag{8}$$

All other subamplitudes are either zero due to the requirements of helicity conservation or can be related by charge conjugation and parity.

3 The Helicity Amplitudes

Figures 1 and 2 show representative box diagrams for the $Hggg$ and $Hgq\bar{q}$ amplitudes, respectively. The Feynman diagrams have been evaluated with the aid of the symbolic manipulation program MAPLE using the straightforward Passarino-Veltman reduction. For the sake of generality, we have regularized the loop integrals by continuing the loop momenta to $(4-2\epsilon)$ dimensions, while taking the number of helicity states of the internal gluons to be $(4-2\,\delta_R\,\epsilon)$. Thus, $\delta_R = 1$ corresponds to the t'Hooft-Veltman scheme [13] and $\delta_R = 0$ corresponds to the four-dimensional-helicity scheme [14].

For the $Hggg$ amplitudes we obtain

$$\begin{aligned}
m^1(1^+,2^+,3^+) &= m^0(1^+,2^+,3^+)\,\frac{\alpha_s}{4\pi}\,r_\Gamma\left(\frac{4\pi\mu^2}{-M_H^2}\right)^\epsilon\Bigg[N_c U \\
&\qquad + \frac{1}{3}(N_c-n_f)\,\frac{S_{31}S_{23}+S_{31}S_{12}+S_{12}S_{23}}{M_H^4}\Bigg] \\
m^1(1^-,2^+,3^+) &= m^0(1^-,2^+,3^+)\,\frac{\alpha_s}{4\pi}\,r_\Gamma\left(\frac{4\pi\mu^2}{-M_H^2}\right)^\epsilon\Bigg[N_c U \\
&\qquad + \frac{1}{3}(N_c-n_f)\,\frac{S_{31}S_{12}}{S_{23}^2}\Bigg]\,,
\end{aligned} \tag{9}$$

where the prefactor is

$$r_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,, \tag{10}$$

and the universal singular factor is

$$\begin{aligned}
U &= \frac{1}{\epsilon^2}\left[-\left(\frac{-M_H^2}{-S_{12}}\right)^\epsilon - \left(\frac{-M_H^2}{-S_{23}}\right)^\epsilon - \left(\frac{-M_H^2}{-S_{31}}\right)^\epsilon\right] + \frac{\pi^2}{2} \\
&\quad - \ln\left(\frac{-S_{12}}{-M_H^2}\right)\ln\left(\frac{-S_{23}}{-M_H^2}\right) - \ln\left(\frac{-S_{12}}{-M_H^2}\right)\ln\left(\frac{-S_{31}}{-M_H^2}\right) - \ln\left(\frac{-S_{23}}{-M_H^2}\right)\ln\left(\frac{-S_{31}}{-M_H^2}\right)
\end{aligned}$$

$$- 2\,\mathrm{Li}_2\Big(1 - \frac{S_{12}}{M_H^2}\Big) \; - \; 2\,\mathrm{Li}_2\Big(1 - \frac{S_{23}}{M_H^2}\Big) \; - \; 2\,\mathrm{Li}_2\Big(1 - \frac{S_{31}}{M_H^2}\Big) \,. \tag{11}$$

For the $Hgq\bar{q}$ amplitude we obtain

$$m^1(g^+, q^-, \bar{q}^+) \;=\; m^0(g^+, q^-, \bar{q}^+)\, \frac{\alpha_s}{4\pi}\, r_\Gamma \left(\frac{4\pi\mu^2}{-M_H^2}\right)^\epsilon \Big[N_c V_1 + \frac{1}{N_c} V_2 + n_f V_3\Big] \,, \tag{12}$$

with

$$\begin{aligned}
V_1 \;=\;& \frac{1}{\epsilon^2}\bigg[-\Big(\frac{-M_H^2}{-S_{gq}}\Big)^\epsilon - \Big(\frac{-M_H^2}{-S_{g\bar{q}}}\Big)^\epsilon\bigg] + \frac{13}{6\epsilon}\Big(\frac{-M_H^2}{-S_{q\bar{q}}}\Big)^\epsilon \\
& - \ln\Big(\frac{-S_{gq}}{-M_H^2}\Big)\ln\Big(\frac{-S_{q\bar{q}}}{-M_H^2}\Big) \; - \; \ln\Big(\frac{-S_{g\bar{q}}}{-M_H^2}\Big)\ln\Big(\frac{-S_{q\bar{q}}}{-M_H^2}\Big) \\
& - 2\,\mathrm{Li}_2\Big(1 - \frac{S_{q\bar{q}}}{M_H^2}\Big) \; - \; \mathrm{Li}_2\Big(1 - \frac{S_{gq}}{M_H^2}\Big) \; - \; \mathrm{Li}_2\Big(1 - \frac{S_{g\bar{q}}}{M_H^2}\Big) \\
& + \frac{83}{18} - \frac{\delta_R}{6} + \frac{\pi^2}{3} - \frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}} \,,
\end{aligned} \tag{13}$$

$$\begin{aligned}
V_2 \;=\;& \Big[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\Big]\Big(\frac{-M_H^2}{-S_{q\bar{q}}}\Big)^\epsilon + \ln\Big(\frac{-S_{gq}}{-M_H^2}\Big)\ln\Big(\frac{-S_{g\bar{q}}}{-M_H^2}\Big) \\
& + \mathrm{Li}_2\Big(1 - \frac{S_{gq}}{M_H^2}\Big) \; + \; \mathrm{Li}_2\Big(1 - \frac{S_{g\bar{q}}}{M_H^2}\Big) \\
& + \frac{7}{2} + \frac{\delta_R}{2} - \frac{\pi^2}{6} - \frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}} \,,
\end{aligned} \tag{14}$$

$$V_3 \;=\; -\frac{2}{3\epsilon}\Big(\frac{-M_H^2}{-S_{q\bar{q}}}\Big)^\epsilon - \frac{10}{9} \,. \tag{15}$$

In these expressions, $N_c = 3$ is the number of colors, n_f is the number of light fermions, and Li_2 is the dilogarithm function. The amplitudes are written for $S_{ij} < 0$ and $M_H^2 < 0$, but they can be analytically continued to the physical region by letting $-S_{ij} = -S_{ij} - i\eta$ and $-M_H^2 = -M_H^2 - i\eta$ for $\eta \to 0^+$. Note that these amplitudes are ultraviolet-unrenormalized amplitudes. Including the renormalization gives the modification

$$m^1 \ \to \ m^1 + (\Delta + 3\delta_g)\, m^0 \ , \tag{16}$$

where Δ is the finite renormalization of the effective Hgg operator, given in eq. (2) and δ_g is the gauge-coupling counterterm. Using the $\overline{\rm MS}$ subtraction scheme, the counterterm is

$$\delta_g \ = \ -\frac{1}{\epsilon}\,\frac{\alpha_s}{4\pi}\,\Gamma(1+\epsilon)\,(4\pi)^\epsilon \left[\frac{11N_c}{6} - \frac{n_f}{3}\right] . \tag{17}$$

The amplitudes satisfy a number of consistency checks. In addition to the previously-mentioned variation of the gauge-fixing condition, we have also shown that the amplitudes are invariant under different choices of reference vectors for the gluon polarizations. The poles in ϵ have been verified to cancel against the dipole subtraction term of Catani and Seymour [15], and the δ_R-dependent terms appear with the correct coefficients to relate the two different regularization schemes [16]. Finally, we have checked that the amplitudes obey the correct one-loop splitting formulae [17] in all of the singular collinear limits.

4 Conclusions

In this paper we have computed the two-loop corrections to the $H \to ggg$ and $H \to gq\bar{q}$ helicity amplitudes in the large-M_t limit. The use of an effective Hgg operator in one-loop Feynman diagrams, along with many of the standard techniques for calculating one-loop QCD amplitudes, has reduced the complexity of this calculation immensely. These amplitudes complete the set needed to perform a NLO calculation of the Higgs $p_\perp$ spectrum in the large-M_t limit, which is currently in progress [18]. In addition, they are part of the set needed for a complete NNLO calculation of the total cross-section in this limit.

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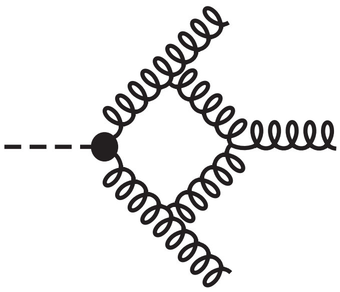

Fig. 1: Box diagram for the $Hggg$ amplitude. The dot represents the effective Hgg vertex in the large-M_t limit.

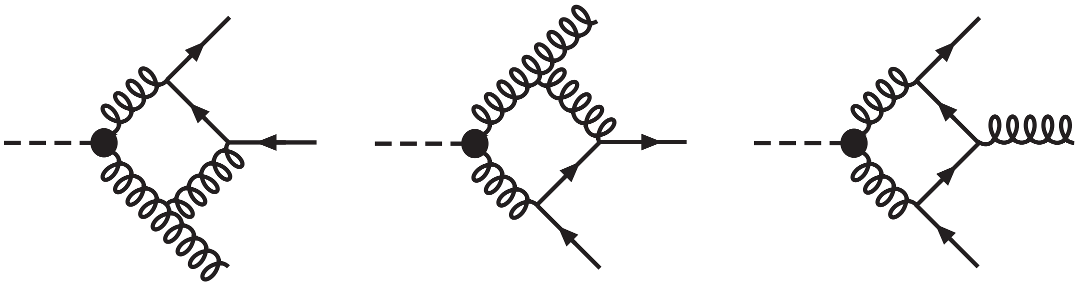

Fig. 2: Box diagrams for the $Hgq\bar{q}$ amplitude. The dot represents the effective Hgg vertex in the large-M_t limit.