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Virtual Next-to-Leading Corrections to the Impact Factors in the High-Energy Limit

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Abstract

We compute the virtual next-to-leading corrections to the impact factors or off-shell coefficient functions in the high-energy limit, in the helicity-amplitude formalism. These agree with previous results by Fadin and collaborators, in the conventional dimensional-regularization scheme. When combined with the known real corrections, these results will provide the complete NLO corrections to the impact factors, which are necessary to use the BFKL resummation at NLL for jet production at both lepton-hadron and hadron-hadron colliders.

1 Introduction

Semi-hard strong-interaction processes, which are characterized by two large and disparate kinematic scales, typically lead to cross sections containing large logarithms. Two examples of this type of process are Deep Inelastic Scattering (DIS) at small x and hadronic dijet production at large rapidity intervals Δy. In DIS the logarithm that appears is $\ln(1/x)$, with $x \simeq Q^2/s$ the squared ratio of the momentum transfer to the photon-hadron center-of-mass energy. In large-rapidity dijet production the large logarithm is $\Delta y \simeq \ln(\hat{s}/|\hat{t}|)$, with $\hat{s}$ the squared parton center-of-mass energy and $|\hat{t}|$ of the order of the squared jet transverse energy. These logarithms will arise in a perturbative calculation at each order in the coupling constant α_s. Alternatively, if the logarithms are large enough, it is preferable to include them through an all-order resummation in the leading logarithmic (LL) approximation performed by means of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [1]-[3].

In order to show how the latter comes about we consider dijet production at large rapidity intervals as a paradigm process. At the lowest order, $\mathcal{O}(\alpha_s^2)$, the underlying parton process is dominated in the $\hat{s} \gg |\hat{t}|$ limit by gluon exchange in the t-channel. Thus the functional form of the amplitudes for gluon-gluon, gluon-quark or quark-quark scattering is the same; they differ only by the color strength in the parton-production vertices. We can then write the cross section for dijet production in the high-energy limit in the following factorized form [4]

$$\frac{d\sigma}{d^2p_{a'\perp}d^2p_{b'\perp}dy_{a'}dy_{b'}} = x_A^0 f_{eff}(x_A^0,\mu_F^2)\, x_B^0 f_{eff}(x_B^0,\mu_F^2)\, \frac{d\hat{\sigma}_{gg}}{d^2p_{a'\perp}d^2p_{b'\perp}}\,, \qquad (1)$$

where a' and b' label the forward and backward outgoing jet, respectively, and the effective *pdf*'s are

$$f_{eff}(x,\mu_F^2) = G(x,\mu_F^2) + \frac{4}{9}\sum_f \left[Q_f(x,\mu_F^2) + \bar{Q}_f(x,\mu_F^2)\right], \qquad (2)$$

where the sum is over the quark flavors. The cross section for gluon-gluon scattering at leading order in the high-energy limit is

$$\frac{d\hat{\sigma}_{gg}^0}{d^2p_{a'\perp}d^2p_{b'\perp}} = \left[\frac{C_A\alpha_s}{p_{a'\perp}^2}\right]\frac{1}{2}\delta^2(p_{a'\perp}+p_{b'\perp})\left[\frac{C_A\alpha_s}{p_{b'\perp}^2}\right], \qquad (3)$$

where the Casimir is $C_A = N_c = 3$.

At higher orders, powers of $\ln(\hat{s}/|\hat{t}|)$ will arise, which can be resummed to all orders in $\alpha_s\ln(\hat{s}/|\hat{t}|)$, *i.e.* to LL accuracy, by the BFKL equation. The factorization formula (1) is unchanged, and the only modification is in the gluon-gluon scattering cross section which becomes [5]

$$\frac{d\hat{\sigma}^0_{gg}}{d^2p_{a'\perp}d^2p_{b'\perp}} \; = \; \left[\frac{C_A\alpha_s}{p^2_{a'\perp}}\right] f(q_{a\perp}, q_{b\perp}, \Delta y) \left[\frac{C_A\alpha_s}{p^2_{b'\perp}}\right], \tag{4}$$

with $\Delta y = y_{a'} - y_{b'}$ and $q_{i\perp}$ the momenta transferred in the t-channel, *i.e.* $q_{a\perp} = p_{a'\perp}$ and $q_{b\perp} = -p_{b'\perp}$. The function $f(q_{a\perp}, q_{b\perp}, \Delta y)$ is the Green's function associated with the gluon exchanged in the t-channel. It is process independent and given in the LL approximation by the solution of the BFKL equation. This equation is a two-dimensional integral equation which describes the evolution in transverse momentum of the gluon propagator exchanged in the t-channel. If we transform to moment space via

$$f(q_{a\perp}, q_{b\perp}, \Delta y) \; = \; \int \frac{d\omega}{2\pi i}\, e^{\omega\Delta y}\, f_\omega(q_{a\perp}, q_{b\perp}) \tag{5}$$

we can write the BFKL equation as

$$\omega\, f_\omega(q_{a\perp}, q_{b\perp}) \; = \frac{1}{2}\,\delta^2(q_{a\perp} - q_{b\perp}) \; + \; \frac{\alpha_s N_c}{\pi^2} \int \frac{d^2k_\perp}{k^2_\perp}\, K(q_{a\perp}, q_{b\perp}, k_\perp)\,, \tag{6}$$

where the kernel K is given by

$$K(q_{a\perp}, q_{b\perp}, k_\perp) = f_\omega(q_{a\perp} + k_\perp, q_{b\perp}) - \frac{q^2_{a\perp}}{k^2_\perp + (q_{a\perp} + k_\perp)^2}\, f_\omega(q_{a\perp}, q_{b\perp})\,. \tag{7}$$

The first term in the kernel accounts for the emission of a real gluon of transverse momentum k and the second term accounts for the virtual radiative corrections. Eq. (6) has been derived in the *multi-Regge kinematics*, which presumes that the produced gluons are strongly ordered in rapidity and have comparable transverse momenta

$$y_{a'} \gg y \gg y_{b'}; \qquad |p_{a'\perp}| \simeq |k_\perp| \simeq |p_{b'\perp}|\,. \tag{8}$$

The solution to the BFKL equation is

$$f(q_{a\perp}, q_{b\perp}, \Delta y) \; = \; \frac{1}{(2\pi)^2 q_{a\perp} q_{b\perp}} \sum_{n=-\infty}^{\infty} e^{in\tilde{\phi}} \int_{-\infty}^{\infty} d\nu\, e^{\omega(\nu,n)\Delta y} \left(\frac{q^2_{a\perp}}{q^2_{b\perp}}\right)^{i\nu}, \tag{9}$$

where $\tilde{\phi}$ is the azimuthal angle between $q_{a\perp}$ and $q_{b\perp}$ and $\omega(\nu, n)$ is the eigenvalue of the BFKL equation whose maximum $\omega(0,0) = 4\ln 2N_c\alpha_s/\pi$ yields the known power-like growth of f in energy [2, 3].

The vertices in square brackets in eq. (4) account for the scattering of an off-shell and an on-shell gluon to produce a gluon $g^*\, g \to g$, and are characteristic of the scattering process under consideration. In the literature they have been called impact factors or LO off-shell coefficient functions. They are known also for photon-photon scattering [3], with impact factor $\gamma g^* \to q\,\bar{q}$; for heavy-quark photoproduction, with impact factor $\gamma g^* \to Q\,\bar{Q}$ [6, 7]; for leptoproduction, with impact factor $\gamma^* g^* \to Q\,\bar{Q}$ [7]; for hadroproduction, with impact factors $gg^* \to Q\,\bar{Q}$ [6] and $g^*\,g^* \to Q\,\bar{Q}$ [7, 8]; for hadroproduction via electroweak boson exchange, with impact factors $V^* g^* \to Q_i\,\bar{Q}_j$, with $V = W, Z$ [9]; for direct photoproduction in hadron-hadron scattering, with impact factor $q\,g^* \to q\,\gamma$ [6]; and for DIS at small x and forward-jet production in DIS, with impact factor $\gamma^* g^* \to q\,\bar{q}$ which may be obtained from $\gamma^* g^* \to Q\,\bar{Q}$ in the massless limit of the heavy quark [10]. In all the cases above the parton cross section is obtained by multiplying the process-independent gluon propagator (9) with the appropriate impact factors. If we label by $\mathcal{F}$ a generic impact factor, then the parton cross section (4) for a generic process in the high-energy limit is

$$\hat{\sigma}_{LL} \sim \mathcal{F}_{LO}(q_{a\perp})\, f_{LL}(q_{a\perp}, q_{b\perp}, \Delta y)\, \mathcal{F}_{LO}(q_{b\perp})\,, \tag{10}$$

where the subscripts stress the accuracy to which the gluon propagator and the impact factors are computed.

The BFKL theory, being a LL resummation and not an exact calculation, makes a few approximations which, even though formally subleading, may be important for any phenomenological purposes:

i) The BFKL resummation is performed at fixed coupling constant, thus any variation in its scale, $\alpha_s(\nu^2) = \alpha_s(\mu^2) - b_0 \ln(\nu^2/\mu^2)\alpha_s^2(\mu^2) + \ldots$, with $b_0 = (11N_c - 2N_f)/12\pi$ and N_f the number of quark flavors, would appear in the next-to-leading-logarithmic (NLL) terms, because it yields terms of $\mathcal{O}(\alpha_s^n \ln(\nu^2/\mu^2) \ln^{n-1}(\hat{s}/|\hat{t}|))$.

ii) From the kinematics of two-parton production at $\hat{s} \gg |\hat{t}|$ we identify the rapidity interval between the tagging jets as $\Delta y \simeq \ln(\hat{s}/|\hat{t}|) \simeq \ln(\hat{s}/k_\perp^2)$, however, we know from the exact kinematics that $\Delta y = \ln(\hat{s}/|\hat{t}| - 1) = \ln(\hat{u}/\hat{t})$ and $|\hat{t}| = k_\perp^2(1 + \exp(-\Delta y))$, therefore the identification of the rapidity interval Δy with $\ln(\hat{s}/|\hat{t}|)$ is correct up to terms of $O(\hat{t}/\hat{s})$.

iii) Because of the strong rapidity ordering (8), there are no collinear divergences in the LL resummation in the BFKL ladder. Jets are determined only to leading order and accordingly have no non-trivial structure.

iv) Finally, energy and longitudinal-momentum are not conserved in the LL limit. Effectively, this means that the momentum fractions $x_{A(B)}$ of the incoming partons are not evaluated exactly, which may induce large numerical errors in certain BFKL predictions. In the exact kinematics, if $n+2$ partons are produced along the ladder, we have

$$x_{A(B)} = \frac{p_{a'\perp}}{\sqrt{s}} e^{(-)y_{a'}} + \sum_{i=1}^{n} \frac{k_{i\perp}}{\sqrt{s}} e^{(-)y_i} + \frac{p_{b'\perp}}{\sqrt{s}} e^{(-)y_{b'}} , \tag{11}$$

where the minus sign in the exponentials of the right-hand side applies to the subscript B on the left-hand side. In the BFKL theory, the LL approximation and the kinematics (8) imply that in the determination of x_A (x_B) in eq. (1) only the first (last) term in eq. (11) is kept,

$$\begin{aligned} x_A^0 &= \frac{p_{a'\perp}}{\sqrt{s}} e^{y_{a'}} , \\ x_B^0 &= \frac{p_{b'\perp}}{\sqrt{s}} e^{-y_{b'}} . \end{aligned} \tag{12}$$

A comparison within dijet production of the exact $\mathcal{O}(\alpha_s^3)$ three-parton production with the truncation of the BFKL ladder to $\mathcal{O}(\alpha_s^3)$ shows that the LL approximation may severely underestimate the exact evaluation of the x's (11), and therefore entail sizable violations of energy-longitudinal momentum conservation [11]. Energy-momentum conservation at each stage in the gluon emission in the BFKL ladder may be achieved through a Monte Carlo implementation of the BFKL equation (6) [12, 13]. However, the weights used to determine the gluon emission and the virtual radiative corrections in the Monte Carlo are still fixed by eq.(7), which is computed using multigluon amplitudes at LL accuracy.

In order to improve on all the points highlighted above the NLL corrections to the BFKL equation need to be calculated. This calculation is close to an end as all the real [14]-[16] and virtual [17]-[19] corrections to the relevant vertices have been computed, and have been assembled to make up the NLL corrections to the BFKL equation [14, 20]. However, in a production cross section the calculation of the process-independent NLL corrections to the gluon propagator exchanged in the t-channel must be matched by impact factors or off-shell coefficient functions computed at the same accuracy. Using the notation of eq. (10),

$$\begin{aligned} \hat{\sigma}_{NLL} \sim\ & \mathcal{F}_{NLO}(q_{a\perp})\, f_{LL}(q_{a\perp}, q_{b\perp}, \Delta y)\, \mathcal{F}_{LO}(q_{b\perp}) \\ +\ & \mathcal{F}_{LO}(q_{a\perp})\, f_{NLL}(q_{a\perp}, q_{b\perp}, \Delta y)\, \mathcal{F}_{LO}(q_{b\perp}) \\ +\ & \mathcal{F}_{LO}(q_{a\perp})\, f_{LL}(q_{a\perp}, q_{b\perp}, \Delta y)\, \mathcal{F}_{NLO}(q_{b\perp}) . \end{aligned} \tag{13}$$

Thus, for each process of interest the corresponding NLO impact factor must be computed.

In this paper we compute the 1-loop corrections to the impact factors. We begin in section 2 by reviewing the LL calculation, as well as the NLO corrections to the impact factors arising from real parton emissions. Throughout this paper we work with fixed helicities to organize the results. In section 3 we turn to the 1-loop virtual corrections to the impact factors. We obtain them from the known 1-loop $g\,g \to g\,g$ and $q\,q \to q\,q$ helicity amplitudes, by expanding in the high-energy limit. Our main results are then the 1-loop corrections to the $g^*\,g \to g$ vertex, eqs. (43) and (50), and to the $g^*\,q \to q$ vertex, eq. (63). These corrections are given in the conventional dimensional regularization (CDR) or t'Hooft-Veltman (HV) schemes and also in the dimensional reduction scheme. They are compared with previous results in the CDR scheme, eqs. (44), (51), and (64), which have been obtained in a different manner. When combined with the known real $\mathcal{O}(\alpha_s)$ corrections, these results provide the complete NLO corrections to the impact factors.

2 Radiative corrections in the high-energy limit

2.1 LL corrections to $\mathcal{O}(\alpha_s^3)$

In the high-energy limit[1] $s \gg |t|$, the amplitude for $g_a\,g_b \to g_{a'}\,g_{b'}$ scattering, with all external gluons outgoing, may be written [1], [21]

$$M^{aa'bb'\,\mathrm{tree}}_{\nu_a\nu_{a'}\nu_{b'}\nu_b} = 2s\left[ig\,f^{aa'c}\,C^{gg(0)}_{-\nu_a\nu_{a'}}(-p_a,p_{a'})\right]\frac{1}{t}\left[ig\,f^{bb'c}\,C^{gg(0)}_{-\nu_b\nu_{b'}}(-p_b,p_{b'})\right], \tag{14}$$

where the ν's label the helicities and the vertices $g^*\,g \to g$ are given by

$$C^{gg(0)}_{-+}(-p_a,p_{a'}) = -1 \qquad C^{gg(0)}_{-+}(-p_b,p_{b'}) = -\frac{p^*_{b'\perp}}{p_{b'\perp}}, \tag{15}$$

with $p_\perp = p_x + ip_y$ the complex transverse momentum. The C-vertices transform into their complex conjugates under helicity reversal, $C^*_{\{\nu\}}(\{k\}) = C_{\{-\nu\}}(\{k\})$. The helicity-flip vertex $C^{(0)}_{++}$ is subleading in the high-energy limit. The square of the amplitude (14), integrated over the phase space, yields the gluon-gluon production rate to leading order, $O(\alpha_s^2)$. For gluon-quark or quark-quark scattering, we only need to exchange the structure constants with color matrices in the fundamental representation and change the vertices $C^{gg(0)}$ to $C^{\bar{q}q(0)}$ [22].

[1]For the remainder of this paper we use s, t, and u without the hat's for the partonic kinematic variables.

Next, we consider the $O(\alpha_s)$ corrections to this process in the high-energy limit. In order to do that, we must consider the emission of an additional gluon, i.e. the $g_a\, g_b \to g_{a'}\, g\, g_{b'}$ scattering amplitude, in the multi-Regge kinematics (8). The scattering amplitude is

$$\begin{aligned} M^{tree}_{gg\to ggg} &= 2s \left[ig\, f^{aa'c}\, C^{gg}_{-\nu_a\nu_{a'}}(-p_a, p_{a'})\right] \frac{1}{t_a} \\ &\times \left[ig\, f^{cdc'}\, C^{g}_{\nu}(q_a, q_b)\right] \frac{1}{t_b} \left[ig\, f^{bb'c'}\, C^{gg}_{-\nu_b\nu_{b'}}(-p_b, p_{b'})\right]\,, \end{aligned} \tag{16}$$

where $t_i \simeq -|q_{i\perp}|^2$ and the Lipatov vertex $g^*\, g^* \to g$ [21, 23], is

$$C^{g}_{+}(q_a, q_b) = \sqrt{2}\, \frac{q^*_{a\perp} q_{b\perp}}{k_\perp}\,. \tag{17}$$

The square of the amplitude (16), integrated over the phase space of the intermediate gluon in multi-Regge kinematics (8) yields an $O(\alpha_s \ln(s/|t|))$ correction to gluon-gluon scattering. This real correction, however, is infrared divergent. To complete the $O(\alpha_s)$ corrections, and to cancel the infrared divergence, we must compute the 1-loop gluon-gluon amplitude in the LL approximation. The virtual radiative corrections to eq. (14) in the LL approximation are obtained, to all orders in α_s, by replacing [1, 24]

$$\frac{1}{t} \to \frac{1}{t}\left(\frac{s}{-t}\right)^{\alpha(t)}\,, \tag{18}$$

in eq. (14), with $\alpha(t)$ related to the loop transverse-momentum integration

$$\alpha(t) \equiv g^2 \alpha^{(1)}(t) = \alpha_s\, N_c\, t \int \frac{d^2 k_\perp}{(2\pi)^2}\, \frac{1}{k^2_\perp (q-k)^2_\perp} \qquad t = q^2 \simeq -q^2_\perp\,, \tag{19}$$

and $\alpha_s = g^2/4\pi$. The infrared divergence in eq. (19) can be regulated in 4 dimensions with an infrared-cutoff mass. Alternatively, using dimensional regularization in $d = 4-2\epsilon$ dimensions, the integral in eq. (19) is performed in $2-2\epsilon$ dimensions, yielding

$$\alpha(t) = g^2 \alpha^{(1)}(t) = 2 g^2\, N_c\, \frac{1}{\epsilon}\left(\frac{\mu^2}{-t}\right)^{\epsilon} c_\Gamma\,, \tag{20}$$

with

$$c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}}\, \frac{\Gamma(1+\epsilon)\, \Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,. \tag{21}$$

Adding the 1-loop gluon-gluon amplitude, multiplied by its tree-level counterpart, to the square of the amplitude (16), integrated over the phase space of the intermediate gluon, cancels the infrared divergences and yields a finite $O(\alpha_s \ln(s/|t|))$ correction to gluon-gluon scattering.

2.2 NLL corrections to $\mathcal{O}(\alpha_s^3)$

In order to compute the real $O(\alpha_s)$ corrections to gluon-gluon scattering which are not accompanied by a $\ln(s/|t|)$, *i.e.* that are NLL, we must relax the strong rapidity ordering between the produced gluons (8). We must allow for the production of two gluons (and to this accuracy also of a $q\bar{q}$ pair) with similar rapidity,

$$\left.\begin{array}{c} y_{a'} \simeq y \gg y_{b'} \\ y_{a'} \gg y \simeq y_{b'} \end{array}\right\} \qquad |p_{a'\perp}| \simeq |k_\perp| \simeq |p_{b'\perp}| \,. \tag{22}$$

The three-gluon production amplitude for the first rapidity ordering is [14, 15, 22]

$$\begin{aligned} & M^{gg}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; k, \nu; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ & \quad = 2s \left\{ C^{g\,g\,g}_{-\nu_a \nu_{a'} \nu}(-p_a, p_{a'}, k) \left[(ig)^2 f^{aa'c} f^{cdc'} \frac{1}{\sqrt{2}} A_{\Sigma\nu_i}(-p_a, p_{a'}, k) + \left(\begin{array}{c} p_{a'} \leftrightarrow k \\ a' \leftrightarrow d \end{array} \right) \right] \right\} \\ & \quad \times \frac{1}{t} \left[ig\, f^{bb'c'} C^{g\,g}_{-\nu_b \nu_{b'}}(-p_b, p_{b'}) \right] , \end{aligned} \tag{23}$$

where we have enclosed the production vertex $g^* g \to g g$ of gluons $p_{a'}$ and k in curly brackets, and with $\sum \nu_i = -\nu_a + \nu_{a'} + \nu$ and

$$\begin{aligned} & C^{g\,g\,g}_{-++}(-p_a, p_{a'}, k) = 1 \,; \qquad C^{g\,g\,g}_{+-+}(-p_a, p_{a'}, k) = \frac{1}{\left(1 + \frac{k^+}{p^+_{a'}}\right)^2} \,; \\ & C^{g\,g\,g}_{++-}(-p_a, p_{a'}, k) = \frac{1}{\left(1 + \frac{p^+_{a'}}{k^+}\right)^2} \,; \qquad A_+(-p_a, p_{a'}, k) = 2\, \frac{p_{b'\perp}}{p_{a'\perp}} \frac{1}{k_\perp - p_{a'\perp} \frac{k^+}{p^+_{a'}}} \,. \end{aligned} \tag{24}$$

The vertex $C^{g\,g\,g}_{+++}(-p_a, p_{a'}, k)$ is subleading to the required accuracy.

The amplitude for the production of a $q\bar{q}$ pair, $g\,g \to \bar{q}qg$, for the first rapidity ordering of eq. (22) is [16, 22],

$$\begin{aligned} & M^{\bar{q}q}(-p_a, -\nu_a; p_{a'}, \nu_{a'}; k, -\nu_{a'}; p_{b'}, \nu_{b'}; -p_b, -\nu_b) \\ & \quad = 2s \left\{ \sqrt{2}\, g^2\, C^{g\,\bar{q}\,q}_{-\nu_a \nu_{a'} -\nu_{a'}}(-p_a, p_{a'}, k) \left[\left(\lambda^{c'} \lambda^a\right)_{d\bar{a'}} A_{-\nu_a}(p_{a'}, k) + \left(\lambda^a \lambda^{c'}\right)_{d\bar{a'}} A_{-\nu_a}(k, p_{a'}) \right] \right\} \\ & \quad \times \frac{1}{t} \left[ig\, f^{bb'c'} C^{g\,g}_{-\nu_b \nu_{b'}}(-p_b, p_{b'}) \right] , \end{aligned} \tag{25}$$

with $p_{a'}$ the antiquark, and the vertex $g^* g \to \bar{q}q$ in curly brackets, with A defined in eq.(24) and $C^{g\,\bar{q}\,q}$ given by,

$$C^{g\,\bar{q}\,q}_{++-}(-p_a, p_{a'}, k) \;=\; \frac{1}{2} \sqrt{\frac{p^+_{a'}}{k^+}} \frac{1}{\left(1 + \frac{p^+_{a'}}{k^+}\right)^2}$$

$$C^{g\bar{q}q}_{+-+}(-p_a, p_{a'}, k) \;=\; \frac{1}{2}\sqrt{\frac{k^+}{p^+_{a'}}}\frac{1}{\left(1+\frac{k^+}{p^+_{a'}}\right)^2}\,. \tag{26}$$

The scattering amplitude $q\,g \to qgg$, from which the vertex $g^*\,q \to g\,q$ is extracted, may be found in ref. [22].

The square of the amplitude (23), integrated over the phase space (22) yields an $O(\alpha_s)$ correction to gluon-gluon scattering. It is, however, infrared divergent, since the vertex A in eq.(24) has a collinear divergence as $2k \cdot p_{a'} \to 0$, and a soft divergence as $k \to 0$. The square of the amplitude (25), integrated over the phase space (22) yields an $O(\alpha_s)$ correction to gluon-gluon scattering, which is only collinearly divergent. In this case the soft divergence of the vertex A in eq.(24) is eliminated by the vanishing of the C-vertices (26) as $k \to 0$, in accordance with the soft quark limit.

We next consider the virtual radiative corrections to the gluon-gluon amplitude. In order to go beyond the LL approximation we need a prescription that allows us to disentangle the virtual corrections to the vertices (15) from the ones that reggeize the gluon (18). Such a prescription is supplied by the general form of the high-energy scattering amplitude, arising from a reggeized gluon in the adjoint representation of $SU(N_c)$ passed in the t-channel[2]. In the helicity basis of eq. (14) this is given by [17]

$$\begin{aligned} M^{aa'bb'}_{\nu_a\nu_{a'}\nu_{b'}\nu_b} \;&=\; s\left[ig\, f^{aa'c}\, C^{gg}_{-\nu_a\nu_{a'}}(-p_a, p_{a'})\right]\frac{1}{t}\left[\left(\frac{s}{-t}\right)^{\alpha(t)} + \left(\frac{-s}{-t}\right)^{\alpha(t)}\right] \\ &\times \left[ig\, f^{bb'c}\, C^{gg}_{-\nu_b\nu_{b'}}(-p_b, p_{b'})\right]\,, \end{aligned} \tag{27}$$

where now

$$\alpha(t) = g^2\alpha^{(1)}(t) + g^4\alpha^{(2)}(t) + O(g^6)\,, \tag{28}$$

and

$$C^{gg} = C^{gg(0)} + g^2 C^{gg(1)} + O(g^4)\,. \tag{29}$$

In the NLL approximation it is necessary to compute $\alpha^{(2)}(t)$ and $C^{gg(1)}$; however, to one loop only $C^{gg(1)}$ appears. In addition, only the dispersive parts of the one-loop amplitude contribute at NLL. Expanding eq. (27) to $O(g^4)$ and using eq. (14), we obtain

$$\begin{aligned} \mathrm{Disp}\, M^{aa'bb'}_{\nu_a\nu_{a'}\nu_{b'}\nu_b} \;&=\; M_4^{\rm tree}\Biggl\{1 \;+\; g^2\Biggl[\alpha^{(1)}(t)\ln\frac{s}{-t} \;+\; \frac{\mathrm{Disp}\, C^{gg(1)}_{-\nu_a\nu_{a'}}(-p_a, p_{a'})}{C^{gg(0)}_{-\nu_a\nu_{a'}}(-p_a, p_{a'})} \\ &\qquad + \frac{\mathrm{Disp}\, C^{gg(1)}_{-\nu_b\nu_{b'}}(-p_b, p_{b'})}{C^{gg(0)}_{-\nu_b\nu_{b'}}(-p_b, p_{b'})}\Biggr]\Biggr\}\,. \end{aligned} \tag{30}$$

[2]Other color structures do occur in the high-energy limit, but they do not contribute at NLL. We show this explicitly for the absorptive part of the 1-loop amplitude in appendix C.

Thus, the NLL corrections to $\mathrm{Disp}\, C^{gg(1)}$ can be extracted directly from the 1-loop $g\, g \to g\, g$ amplitude. We shall compute these virtual corrections to the vertices (15) in sect. 3.

Finally, the NLO impact factors are obtained by combining the square of the vertices $g^*\, g \to g\, g$ (24) and $g^*\, g \to \bar{q} q$ (26), integrated over the phase space of the intermediate gluon, with the gluon-loop and quark-loop contributions to $\mathrm{Disp}\, C^{gg(1)}$, respectively.

3 The 1-loop corrections to the C vertices

3.1 The 1-loop four-gluon amplitude

The color decomposition of a tree-level multigluon amplitude in a helicity basis is [25]

$$M_n^{tree} = 2^{n/2}\, g^{n-2} \sum_{S_n/Z_n} \mathrm{tr}(\lambda^{d_{\sigma(1)}} \cdots \lambda^{d_{\sigma(n)}})\, m_n(p_{\sigma(1)}, \nu_{\sigma(1)}; ...; p_{\sigma(n)}, \nu_{\sigma(n)}) \,, \tag{31}$$

where $d_1, ..., d_n$, and $\nu_1, ..., \nu_n$ are respectively the colors and the polarizations of the gluons, the λ's are the color matrices[3] in the fundamental representation of $\mathrm{SU}(N_c)$ and the sum is over the noncyclic permutations S_n/Z_n of the set $[1, ..., n]$. We take all the momenta as outgoing, and consider the *maximally helicity-violating* configurations $(-, -, +, ..., +)$ for which the gauge-invariant subamplitudes, $m_n(p_1, \nu_1; ...; p_n, \nu_n)$, assume the form [25],

$$m_n(-, -, +, ..., +) = \frac{\langle p_i p_j \rangle^4}{\langle p_1 p_2 \rangle \cdots \langle p_{n-1} p_n \rangle \langle p_n p_1 \rangle} \,, \tag{32}$$

where i and j are the gluons of negative helicity. The configurations $(+, +, -, ..., -)$ are then obtained by replacing the $\langle pk \rangle$ products with $[kp]$ products. We give the formulae for these spinor products in appendix A. Using the high-energy limit of the spinor products (74), the tree-level amplitude for $g\, g \to g\, g$ scattering may be cast in the form (14).

The color decomposition of one-loop multigluon amplitudes is also known [26]. For four gluons it is,

$$\begin{aligned} M_4^{1-loop} &= 4g^4 \Bigg[\sum_{S_4/Z_4} \mathrm{tr}(\lambda^{d_{\sigma(1)}} \lambda^{d_{\sigma(2)}} \lambda^{d_{\sigma(3)}} \lambda^{d_{\sigma(4)}})\, m_{4:1}(\sigma(1), \sigma(2), \sigma(3), \sigma(4)) \\ &\quad + \sum_{S_4/Z_2^3} \mathrm{tr}(\lambda^{d_{\sigma(1)}} \lambda^{d_{\sigma(2)}}) \mathrm{tr}(\lambda^{d_{\sigma(3)}} \lambda^{d_{\sigma(4)}})\, m_{4:3}(\sigma(1), \sigma(2), \sigma(3), \sigma(4)) \Bigg] \,, \end{aligned} \tag{33}$$

[3] Note that eq.(31) differs by the $2^{n/2}$ factor from the expression given in ref.[25], because we use the standard normalization of the λ matrices, $\mathrm{tr}(\lambda^a \lambda^b) = \delta^{ab}/2$.

where $\sigma(i)$ is a shorthand for $p_{\sigma(i)}, \nu_{\sigma(i)}$ in the subamplitudes. In the second line the sum is over the permutations of the four color indices, up to permutations within each trace and to permutations which interchange the two traces. There are two independent helicity-conserving configurations $(-,-,+,+)$ and $(-,+,-,+)$, for which the subamplitudes of the type $m_{4:1}$ are [27], [28],

$$\begin{aligned} m_{4:1}(-,-,+,+) &= m_4(-,-,+,+)\, c_\Gamma \\ &\times \left\{ \left(-\frac{\mu^2}{s_{14}}\right)^\epsilon \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon} \ln\frac{s_{12}}{s_{14}} - \frac{64}{9} - \frac{\delta_R}{3} + \pi^2 \right) \right.\right. \\ &\left.\left. + N_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right] - \frac{\beta_0}{\epsilon} \right\} \end{aligned} \tag{34}$$

$$\begin{aligned} m_{4:1}(-,+,-,+) &= m_4(-,+,-,+)\, c_\Gamma \\ &\times \left\{ \left(-\frac{\mu^2}{s_{14}}\right)^\epsilon \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon} \ln\frac{s_{12}}{s_{14}} - \frac{64}{9} - \frac{\delta_R}{3} + \pi^2 \right) + N_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right.\right. \\ &- (N_c - N_f) \frac{s_{12}s_{14}}{s_{13}^2} \left[1 - \left(\frac{s_{12}}{s_{13}} - \frac{s_{14}}{s_{13}} \right) \ln\frac{s_{14}}{s_{12}} - \left(\frac{s_{12}s_{14}}{s_{13}^2} - 2 \right) \left(\ln^2\frac{s_{14}}{s_{12}} + \pi^2 \right) \right] \\ &\left.\left. + \beta_0 \frac{s_{14}}{s_{13}} \ln\frac{s_{14}}{s_{12}} - \frac{3}{2} N_f \frac{s_{12}s_{14}}{s_{13}^2} \left(\ln^2\frac{s_{14}}{s_{12}} + \pi^2 \right) \right] - \frac{\beta_0}{\epsilon} \right\}, \end{aligned} \tag{35}$$

with N_f the number of quark flavors, c_Γ given in eq. (21), $\beta_0 = (11N_c - 2N_f)/3$,

$$\delta_R = \begin{cases} 1 & \text{HV or CDR scheme,} \\ 0 & \text{dimensional reduction scheme,} \end{cases} \tag{36}$$

the tree amplitude m_4 given in eq. (32), and the $\overline{\text{MS}}$ ultraviolet counterterm in the last term of eq. (34) and (35).

There are three subamplitudes of the type $m_{4:3}$ to be determined in the second line of eq. (33): $m_{4:3}(1,2,3,4)$, $m_{4:3}(1,3,2,4)$ and $m_{4:3}(1,4,2,3)$. However, any subamplitude of the type $m_{4:3}$ may be obtained from the subamplitudes of type $m_{4:1}$ [26]. They satisfy

$$m_{4:3}(1,2,3,4) = \frac{1}{N_c} \sum_{S_4/Z_4} m_{4:1}(1,2,3,4)\ , \tag{37}$$

where only the N_f-independent, unrenormalized contributions to $m_{4:1}$ are included in this formula [28, 29]. Then we have

$$m_{4:3}(1,2,3,4) = m_{4:3}(1,3,2,4) = m_{4:3}(1,4,2,3)\ , \tag{38}$$

and it suffices to determine $m_{4:3}(1,2,3,4)$.

To obtain the next-to-leading log corrections to the helicity-conserving $g^* g \to g$ vertex, we need the amplitude $M_4^{1-loop}(B-, A-, A'+, B'+)$ in the high-energy limit. We must consider each of the color orderings in eq. (33) and expand the expressions (34) and (35) in powers of t/s, retaining only the leading power, which yields the leading and next-to-leading terms in $\ln(s/t)$. In fact, at next-to-leading log, we only need to keep the dispersive parts of the subamplitudes $m_{4:1}$ and $m_{4:3}$. It is not difficult to show that if a given color ordering of m_4 is suppressed by a power of t/s at tree-level, then the corresponding color ordering of $m_{4:1}$ will also be suppressed at one-loop. Then the leading color orderings of type $m_{4:1}(-,-,+,+)$ (as in eq. (34)) occur with $s_{12} = s, s_{14} = t$. Thus, we have

$$\begin{aligned} \mathrm{Disp}\, m_{4:1}(-,-,+,+) &= m_4(-,-,+,+)\, c_\Gamma \\ &\times \left\{ \left(\frac{\mu^2}{-t}\right)^\epsilon \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon} \ln \frac{s}{-t} - \frac{64}{9} - \frac{\delta_R}{3} + \pi^2 \right) \right.\right. \\ &\left.\left. + N_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right] - \frac{\beta_0}{\epsilon} \right\} . \end{aligned} \tag{39}$$

The leading color orderings of type $m_{4:1}(-,+,-,+)$ (as in eq. (35)) occur with $s_{14} = t, s_{12} = u$ or $s_{14} = u, s_{12} = t$. However, eq. (35) is symmetric in s_{14} and s_{12} up to $\mathcal{O}(\epsilon)$, so both orderings give the same result[4]. Using $u = -s - t$, we see that eq. (39) holds for the dispersive parts of $m_{4:1}(-,+,-,+)$ as well. Finally, the subamplitude $m_{4:3}$ can be obtained using eq. (39) and (37). We find that Disp $m_{4:3}$ vanishes to power accuracy in s/t:

$$\mathrm{Disp}\, m_{4:3}(B-, A-, A'+, B'+) = 0 + O(t/s)\,. \tag{40}$$

The other color orderings of $m_{4:3}$ also vanish due to (38). Thus, we conclude that the dispersive part of the one-loop amplitude is simply proportional to the tree amplitude to leading power in t/s:

$$\begin{aligned} \mathrm{Disp}\, M_4^{1-loop}(B-, A-, A'+, B'+) &= M_4^{tree}(B-, A-, A'+, B'+)\, g^2\, c_\Gamma \\ &\times \left\{ \left(\frac{\mu^2}{-t}\right)^\epsilon \left[N_c \left(-\frac{4}{\epsilon^2} - \frac{11}{3\epsilon} + \frac{2}{\epsilon} \ln \frac{s}{-t} - \frac{64}{9} - \frac{\delta_R}{3} + \pi^2 \right) \right.\right. \\ &\left.\left. + N_f \left(\frac{2}{3\epsilon} + \frac{10}{9} \right) \right] - \frac{\beta_0}{\epsilon} \right\} . \end{aligned} \tag{41}$$

[4]Using the reflection and cyclic symmetries of the subamplitudes [26], we see that $m_{4:1}(A-, A'+, B-, B'+) = m_{4:1}(A-, B'+, B-, A'+)$, i.e. if eq. (35) were calculated to all orders in ϵ it would be exactly symmetric in s_{14} and s_{12}.

To LL accuracy, using eq. (20), we find that eq. (41) reduces to

$$\mathrm{Disp}\, M_4^{1-loop}(B-, A-, A'+, B'+) = g^2 \alpha^{(1)}(t) \,\ln \frac{s}{-t}\, M_4^{\rm tree}\,, \tag{42}$$

in agreement with eq. (30). To NLL accuracy, we may extract from eq. (30) and (41) the 1-loop corrections to the helicity-conserving vertex[5] $g^*\, g \to g$,

$$\begin{aligned}
&\frac{\mathrm{Disp}\, C^{gg(1)}_{-+}(-p_a, p_{a'})}{C^{gg(0)}_{-+}(-p_a, p_{a'})} = \frac{\mathrm{Disp}\, C^{gg(1)}_{-+}(-p_b, p_{b'})}{C^{gg(0)}_{-+}(-p_b, p_{b'})} = \\
&\qquad c_\Gamma \left\{ \left(\frac{\mu^2}{-t}\right)^\epsilon \left[N_c \left(-\frac{2}{\epsilon^2} - \frac{11}{6\epsilon} - \frac{32}{9} - \frac{\delta_R}{6} + \frac{\pi^2}{2} \right) + N_f \left(\frac{1}{3\epsilon} + \frac{5}{9} \right) \right] - \frac{\beta_0}{2\epsilon} \right\}\,.
\end{aligned} \tag{43}$$

We can compare this to the unrenormalized one-loop corrections to the helicity-conserving vertex calculated in ref. [17] and [30][6]. Rewriting it as

$$\begin{aligned}
\Gamma^{(1)}_{-+}(t) &= c_\Gamma \left(\frac{\mu^2}{-t}\right)^\epsilon \frac{1}{\epsilon(1-2\epsilon)} \left\{ N_f \frac{1-\epsilon}{3-2\epsilon} \right. \\
&\quad \left. + N_c \left[(1-2\epsilon)[\psi(1+\epsilon) - 2\psi(-\epsilon) + \psi(1)] - \frac{1}{4(3-2\epsilon)} - \frac{7}{4} \right] \right\}\,,
\end{aligned} \tag{44}$$

and expanding to $O(\epsilon^0)$, we see that it agrees with eq. (43), with $\delta_R = 1$.

At one-loop there are also contributions to the helicity-violating part of the vertex $C^{gg(1)}$. To calculate this we need the subamplitudes [27], [28],

$$m_{4:1}(-,+,+,+) = \frac{1}{48\pi^2} (N_c - N_f) \frac{[24]^2}{[12]\langle 23\rangle\langle 34\rangle[41]} (s_{12} + s_{14}) \tag{45}$$

$$m_{4:3}(-,+,+,+) = \frac{1}{8\pi^2} \frac{\langle 12\rangle[24]\langle 41\rangle}{\langle 23\rangle\langle 34\rangle\langle 24\rangle}\,. \tag{46}$$

Using eq. (45) and the spinor products (74), we find in the high-energy limit,

$$\begin{aligned}
&m_{4:1}\left(\sigma(B-), \sigma(A+), \sigma(A'+), \sigma(B'+)\right) \\
&\qquad = -\frac{1}{48\pi^2} (N_c - N_f) \frac{p^*_{b'\perp}}{p_{b'\perp}}\, m_4\left(\sigma(B-), \sigma(A-), \sigma(A'+), \sigma(B'+)\right)\,,
\end{aligned} \tag{47}$$

[5]Of course, there is no gauge-invariant way of distinguishing the contribution to $C^{gg(1)}_{-\nu_a\nu_{a'}}(-p_a, p_{a'})$ from the one to $C^{gg(1)}_{-\nu_b\nu_{b'}}(-p_b, p_{b'})$, so we conventionally assume that they are equal.

[6]The purely gluonic part of the unrenormalized 1-loop corrections in ref. [17] is marred by misprints and mistakes. The correct expression is given in ref. [31].

where $(\sigma(B), \sigma(A), \sigma(A'), \sigma(B'))$ spans the four permutations: (B, A, A', B'), (B, B', A, A'), (B, A', A, B') and (B, B', A', A), which yield the leading color orderings at tree level [21]. The other color orderings are subleading and, using eq. (46), we find that $m_{4:3}(-,+,+,+)$ is also subleading in the high-energy limit. Therefore we have,

$$M_4^{1-loop}(B-, A+, A'+, B'+) = -\frac{g^2}{48\pi^2}\,(N_c - N_f)\,\frac{p^*_{b'\perp}}{p_{b'\perp}}\,M_4^{tree}(B-, A-, A'+, B'+)\,. \quad (48)$$

From eq. (14), and (27) expanded to $O(g^4)$, with $C^{gg(0)}_{++} = 0$, we may write,

$$M_4^{1-loop}(B-, A+, A'+, B'+) = g^2\frac{C^{gg(1)}_{++}(-p_a, p_{a'})}{C^{gg(0)}_{-+}(-p_a, p_{a'})}\,M_4^{tree}(B-, A-, A'+, B'+)\,, \quad (49)$$

and comparing eq. (49) to (48), we obtain

$$\frac{C^{gg(1)}_{++}(-p_a, p_{a'})}{C^{gg(0)}_{-+}(-p_a, p_{a'})} = -\frac{1}{48\pi^2}\,(N_c - N_f)\,\frac{p^*_{b'\perp}}{p_{b'\perp}}\,. \quad (50)$$

Using the amplitude $M_4^{1-loop}(B-, A-, A'-, B'+)$, it is possible to check that the 1-loop corrections maintain the property of complex conjugation under helicity reversal, i.e. that $[C^{gg(1)}_{++}]^* = C^{gg(1)}_{--}$. The helicity-violating part of the vertex has been also computed in ref.[17], [30]

$$\Gamma^{(1)}_{++}(t) = c_\Gamma\left(\frac{\mu^2}{-t}\right)^\epsilon \frac{1}{(3-2\epsilon)(1-2\epsilon)}\left(N_c - \frac{N_f}{1-\epsilon}\right)\,, \quad (51)$$

which, when expanded to $O(\epsilon^0)$, coincides with eq. (50) up to an irrelevant overall phase.

3.2 The 1-loop four-quark amplitude

The tree-level amplitude for $q\,q \to q\,q$ scattering in the high-energy limit is

$$M^{\bar{a}a'\bar{b}b'\ \mathrm{tree}}_{\nu_a\nu_b} = 2s\left[g\,\lambda^c_{a'\bar{a}}\,C^{\bar{q}q(0)}_{-\nu_a\nu_a}(-p_a, p_{a'})\right]\frac{1}{t}\left[g\,\lambda^c_{b'\bar{b}}\,C^{\bar{q}q(0)}_{-\nu_b\nu_b}(-p_b, p_{b'})\right]\,, \quad (52)$$

where we have labelled the incoming quarks as outgoing antiquarks by convention. The C-vertices $g^*\,q \to q$ are

$$C^{\bar{q}q(0)}_{-+}(-p_a, p_{a'}) = -1\,; \qquad C^{\bar{q}q(0)}_{-+}(-p_b, p_{b'}) = -\left(\frac{p^*_{b'\perp}}{p_{b'\perp}}\right)^{1/2}\,, \quad (53)$$

and we note that helicity is exactly conserved on a massless fermion line. The tree-level high-energy limit for quark-quark scattering (52) is similar to that for gluon-gluon scattering (14), since both of them are dominated by gluon exchange in the t-channel. This suggests that the virtual radiative corrections to eq. (52) in the LL approximation can be obtained by Reggeizing the t-channel gluon as in eq. (18), and that the general form of the quark-quark scattering amplitude in the high-energy limit is essentially the same as that for gluon-gluon scattering (27). Accordingly, its expansion to $O(g^4)$ yields

$$\begin{aligned} \mathrm{Disp}\, M^{\bar{a}a'\bar{b}b'}_{\nu_a\nu_b} &= M_4^{\rm tree}\Biggl\{1 + g^2\biggl[\alpha^{(1)}(t)\ln\frac{s}{-t} + \frac{\mathrm{Disp}\, C^{\bar{q}q(1)}_{-\nu_a\nu_a}(-p_a,p_{a'})}{C^{\bar{q}q(0)}_{-\nu_a\nu_a}(-p_a,p_{a'})} \\ &\qquad + \frac{\mathrm{Disp}\, C^{\bar{q}q(1)}_{-\nu_b\nu_b}(-p_b,p_{b'})}{C^{\bar{q}q(0)}_{-\nu_b\nu_b}(-p_b,p_{b'})}\biggr]\Biggr\}, \end{aligned} \tag{54}$$

with $M_4^{\rm tree}$ in eq. (52).

The color decomposition of the quark-quark scattering amplitude is respectively at tree level [25]

$$M_4^{tree}(\bar{q},\bar{Q};Q,q) = g^2\left(\delta_{i_1i_3}\delta_{i_2i_4} - \frac{1}{N_c}\delta_{i_1i_4}\delta_{i_2i_3}\right) a_4(1,2;3,4)\,, \tag{55}$$

and at one loop [28]

$$\begin{aligned} M_4^{1-loop}(\bar{q},\bar{Q};Q,q) &= g^4\left[\left(\delta_{i_1i_3}\delta_{i_2i_4} - \frac{1}{N_c}\delta_{i_1i_4}\delta_{i_2i_3}\right) a_{4:1}(1,2;3,4)\right. \\ &+ \left.\delta_{i_1i_3}\delta_{i_2i_4} a_{4:2}(1,2;3,4)\right]\,. \end{aligned} \tag{56}$$

The tree-level subamplitude is

$$a_4(-,-;+,+) = \frac{\langle 12\rangle[34]}{s_{14}}\,. \tag{57}$$

Using the spinor products (74) in the high-energy limit in this equation and substituting into eq. (55), we obtain eq. (52).

The one-loop subamplitudes are [28]

$$\begin{aligned} & a_{4:1}(-,-;+,+) = a_4(-,-;+,+)c_\Gamma \\ & \quad\times\left\{\left(-\frac{\mu^2}{s_{14}}\right)^\epsilon\left[N_c\left(-\frac{2}{\epsilon^2}+\frac{2}{3\epsilon}+\frac{2}{\epsilon}\ln\frac{s_{12}}{s_{14}}+\frac{19}{9}-\frac{2\delta_R}{3}+\pi^2\right)-N_f\left(\frac{2}{3\epsilon}+\frac{10}{9}\right)\right.\right. \\ & \quad+\frac{1}{N_c}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+\frac{2}{\epsilon}\ln\frac{s_{12}}{s_{13}}+7+\delta_R-\frac{1}{2}\frac{s_{14}}{s_{12}}\left(1-\frac{s_{13}}{s_{12}}\right)\left(\ln^2\frac{s_{14}}{s_{13}}+\pi^2\right)\right. \end{aligned}$$

$$-\frac{s_{14}}{s_{12}}\ln\frac{s_{14}}{s_{13}}\Big)\Big]-\frac{\beta_0}{\epsilon}\Bigg\} \tag{58}$$

$$\begin{aligned}a_{4:2}(-,-;+,+) &= a_4(-,-;+,+)c_\Gamma \\ &\times\left(-\frac{\mu^2}{s_{14}}\right)^\epsilon\frac{N_c^2-1}{N_c}\left[-\frac{2}{\epsilon}\ln\frac{s_{12}}{s_{13}}+\frac{1}{2}\frac{s_{14}}{s_{12}}\left(1-\frac{s_{13}}{s_{12}}\right)\left(\ln^2\frac{s_{14}}{s_{13}}+\pi^2\right)\right. \\ &\left.+\frac{s_{14}}{s_{12}}\ln\frac{s_{14}}{s_{13}}\right] .\end{aligned} \tag{59}$$

As in sect. 3.1, we only consider the dispersive part of these equations. Expanding in powers of t/s and retaining only the leading power yields the LL and NLL terms in $\ln(s/t)$:

$$\begin{aligned}\mathrm{Disp}\, a_{4:1}(B-,A-;A'+,B'+) &= a_4(B-,A-;A'+,B'+)c_\Gamma \\ &\times\left\{\left(-\frac{\mu^2}{t}\right)^\epsilon\left[N_c\left(-\frac{2}{\epsilon^2}+\frac{2}{3\epsilon}+\frac{2}{\epsilon}\ln\frac{s}{-t}+\frac{19}{9}-\frac{2\delta_R}{3}+\pi^2\right)\right.\right. \\ &\left.\left.-N_f\left(\frac{2}{3\epsilon}+\frac{10}{9}\right)+\frac{1}{N_c}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+7+\delta_R\right)\right]-\frac{\beta_0}{\epsilon}\right\} .\end{aligned} \tag{60}$$

The dispersive part of $a_{4:2}$ is subleading to power accuracy, and we neglect it. Substituting eq. (60) into eq. (56) and using eq. (55), we obtain for the configuration $(B-,A-;A'+,B'+)$:

$$\begin{aligned}\mathrm{Disp}\, M_4^{1-loop}(\bar q,\bar Q,Q,q) &= M_4^{tree}(\bar q,\bar Q,Q,q)g^2c_\Gamma \\ &\times\left\{\left(-\frac{\mu^2}{t}\right)^\epsilon\left[N_c\left(-\frac{2}{\epsilon^2}+\frac{2}{3\epsilon}+\frac{2}{\epsilon}\ln\frac{s}{-t}+\frac{19}{9}-\frac{2\delta_R}{3}+\pi^2\right)\right.\right. \\ &\left.\left.-N_f\left(\frac{2}{3\epsilon}+\frac{10}{9}\right)+\frac{1}{N_c}\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}+7+\delta_R\right)\right]-\frac{\beta_0}{\epsilon}\right\} .\end{aligned} \tag{61}$$

To LL accuracy, eq. (61) reduces to

$$\mathrm{Disp}\, M_4^{1-loop}(B-,A-;A'+,B'+) = g^2\alpha^{(1)}(t)\,\ln\frac{s}{-t}\,M_4^{\rm tree}\,, \tag{62}$$

which is exactly the same form as eq. (42), due to the universality of the LL contribution. Comparing the expansion (54) to eq. (61), we obtain the 1-loop correction to the C-vertex $g^*\,q\to q$,

$$\begin{aligned}\frac{\mathrm{Disp}\, C_{-+}^{\bar qq(1)}(-p_a,p_{a'})}{C_{-+}^{\bar qq(0)}(-p_a,p_{a'})} &= \frac{\mathrm{Disp}\, C_{-+}^{\bar qq(1)}(-p_b,p_{b'})}{C_{-+}^{\bar qq(0)}(-p_b,p_{b'})} = \\ &c_\Gamma\left\{\left(-\frac{\mu^2}{t}\right)^\epsilon\left[N_c\left(-\frac{1}{\epsilon^2}+\frac{1}{3\epsilon}+\frac{19}{18}-\frac{\delta_R}{3}+\frac{\pi^2}{2}\right)\right.\right. \\ &\left.\left.-N_f\left(\frac{1}{3\epsilon}+\frac{5}{9}\right)+\frac{1}{N_c}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}+\frac{7}{2}+\frac{\delta_R}{2}\right)\right]-\frac{\beta_0}{2\epsilon}\right\} .\end{aligned} \tag{63}$$

The unrenormalized 1-loop corrections to the C-vertex $g^*\,q \to q$ have also been computed in ref. [32],

$$\begin{aligned}\Gamma^{(1)}_{-+q}(t) &= c_\Gamma \left(\frac{\mu^2}{-t}\right)^\epsilon \frac{1}{\epsilon(1-2\epsilon)} \left\{ -N_f \frac{1-\epsilon}{3-2\epsilon} + \frac{1}{N_c}\left(\frac{1}{\epsilon} - \frac{1-2\epsilon}{2}\right) \right. \\ &\quad \left. + N_c \left[(1-2\epsilon)[\psi(1+\epsilon) - 2\psi(-\epsilon) + \psi(1)] + \frac{1}{4(3-2\epsilon)} + \frac{1}{\epsilon} - \frac{7}{4} \right] \right\} . \quad (64)\end{aligned}$$

Expanding this to $O(\epsilon^0)$, we see that it agrees with eq. (63), with $\delta_R = 1$.

4 Conclusions

Our main results are the 1-loop corrections to the $g^*\,g \to g$ vertex, eqs. (43) and (50), and to the $g^*\,q \to q$ vertex, eq. (63). The dispersive parts, which are all that is needed at NLL, agree with previous calculations performed in the CDR scheme [31], which have been obtained in a different manner. We also give the absorptive parts in appendix C. The virtual corrections, when combined with the $g^*\,g \to g\,g$ (eq. (23)), $g^*\,g \to q\,\bar{q}$ (eq. (25)), and $g^*\,q \to g\,q$ [22] real corrections, give the NLO gluon and quark impact factors, which are necessary to use the BFKL resummation at NLL for jet production at both lepton-hadron and hadron-hadron colliders.

The NLL corrections to the BFKL approximation are greatly desired, because they incorporate several physical effects which are lacking in the LL resummation. For instance, the real corrections to the impact factor contain a collinear singularity, as explicitly seen in eq. (24). Thus, it is possible only beyond LL to consider effects such as jet definition and cone-size dependence of the cross section. In addition, the NLL corrections also improve the errors due to energy and longitudinal momentum conservation. Although these effects can be included in the parton density functions through a BFKL Monte Carlo simulation [12, 13], there are still large uncertainties due to the LL approximation used for the matrix elements. The NLL corrections should reduce these uncertainties substantially.

Finally, the incorporation of renormalization- and factorization-scale dependence only becomes manifest at NLL. Note that the virtual correction to the $g^*\,g \to g$ vertex (43) can be written

$$\frac{\mathrm{Disp}\, C^{gg(1)}_{-+}}{C^{gg(0)}_{-+}} = c_\Gamma \left(\frac{\mu^2}{-t}\right)^\epsilon \left[-\frac{1}{2}\beta_0 \ln\frac{-t}{\mu^2} \; + \; \text{non-log terms} \right] . \quad (65)$$

Thus, the scale dependence of the amplitude can be obtained by simply replacing $g(\mu^2) \to g(-t) = g(\mu^2)\,[1 - (\beta_0\,\alpha_s/8\pi)\ln(-t/\mu^2) + \ldots]$. Of course, this is to be expected, since there is only one scale, $-t \simeq q_\perp^2$, which enters the impact factor. However, in the BFKL kernel, there are several scales which enter the vertex $g^*(q_\perp)\, g^*(q_\perp + k_\perp) \to g(k_\perp)$. At LL these scales are all assumed comparable, and cannot be distinguished as far as the choice of renormalization scale. By studying the NLL corrections to this vertex [17, 33], which can be obtained from the 1-loop five-gluon amplitudes [29], one can investigate the proper scale-dependence of the BFKL kernel.

A Spinor Algebra in the Multi-Regge kinematics

We consider the scattering of two gluons of momenta p_a and p_b into $n+2$ gluons of momenta p_i, where $i = a', b', 1 \ldots n$. Using light-cone coordinates $p^\pm = p_0 \pm p_z$, and complex transverse coordinates $p_\perp = p_x + ip_y$, with scalar product $2p \cdot q = p^+q^- + p^-q^+ - p_\perp q_\perp^* - p_\perp^* q_\perp$, the gluon 4-momenta are,

$$
\begin{aligned}
p_a &= \left(p_a^+, 0; 0, 0\right) , \\
p_b &= \left(0, p_b^-; 0, 0\right) , \\
p_i &= \left(|p_{i\perp}| e^{y_i}, |p_{i\perp}| e^{-y_i}; |p_{i\perp}| \cos\phi_i, |p_{i\perp}| \sin\phi_i\right) ,
\end{aligned}
\tag{66}
$$

where to the left of the semicolon we have the + and - components, and to the right the transverse components. y is the gluon rapidity and ϕ is the azimuthal angle between the vector $p_\perp$ and an arbitrary vector in the transverse plane. Momentum conservation gives

$$
\begin{aligned}
0 &= \sum p_{i\perp} , \\
p_a^+ &= \sum p_i^+ , \\
p_b^- &= \sum p_i^- .
\end{aligned}
\tag{67}
$$

For each massless momentum p there is a positive and negative helicity spinor, $|p+\rangle$ and $|p-\rangle$, so we can consider two types of spinor products

$$
\begin{aligned}
\langle pq \rangle &= \langle p - | q + \rangle \\
[pq] &= \langle p + | q - \rangle .
\end{aligned}
\tag{68}
$$

Phases are chosen so that $\langle pq \rangle = -\langle qp \rangle$ and $[pq] = -[qp]$. For the momentum under consideration the spinor products are

$$
\langle p_i p_j \rangle = p_{i\perp} \sqrt{\frac{p_j^+}{p_i^+}} - p_{j\perp} \sqrt{\frac{p_i^+}{p_j^+}} ,
$$

$$
\begin{aligned}
\langle p_a p_i \rangle &= -\sqrt{\frac{p_a^+}{p_i^+}}\, p_{i\perp}\,, \\
\langle p_i p_b \rangle &= -\sqrt{p_i^+ p_b^-}\,, \\
\langle p_a p_b \rangle &= -\sqrt{p_a^+ p_b^-} = -\sqrt{s_{ab}}\,,
\end{aligned}
\tag{69}
$$

where we have used the mass-shell condition $|p_{i\perp}|^2 = p_i^+ p_i^-$. The other type of spinor product can be obtained from

$$
[pq] = \pm\langle qp\rangle^*\,, \tag{70}
$$

where the $+$ is used if p and q are both ingoing or both outgoing, and the $-$ is used if one is ingoing and the other outgoing.

In the multi-Regge kinematics, the gluons are strongly ordered in rapidity and have comparable transverse momentum:

$$
y_{a'} \gg y_1 \gg \ldots y_n \gg y_{b'}; \qquad |p_{i\perp}| \simeq |p_\perp|\,. \tag{71}
$$

Then the momentum conservation (67) in the $\pm$ directions reduces to

$$
\begin{aligned}
p_a^+ &\simeq p_{a'}^+\,, \\
p_b^- &\simeq p_{b'}^-\,,
\end{aligned}
\tag{72}
$$

and the Mandelstam invariants become

$$
\begin{aligned}
s_{ab} &= 2p_a \cdot p_b \simeq p_{a'}^+ p_{b'}^- \\
s_{ai} &= -2p_a \cdot p_i \simeq -p_{a'}^+ p_i^- \\
s_{bi} &= -2p_b \cdot p_i \simeq -p_i^+ p_{b'}^- \\
s_{ij} &= 2p_i \cdot p_j \simeq |p_{i\perp}||p_{j\perp}| e^{y_i - y_j} = p_i^+ p_j^- \qquad (y_i \gg y_j)\,,
\end{aligned}
\tag{73}
$$

where $i, j = a', b', 1 \ldots n$. In this limit the spinor products (69) become

$$
\begin{aligned}
\langle p_a p_b \rangle &\simeq \langle p_{a'} p_b \rangle \simeq -\sqrt{\frac{p_{a'}^+}{p_{b'}^+}}\,|p_{b'\perp}| \\
\langle p_a p_{b'} \rangle &\simeq \langle p_{a'} p_{b'} \rangle = -\sqrt{\frac{p_{a'}^+}{p_{b'}^+}}\, p_{b'\perp} \\
\langle p_a p_{a'} \rangle &\simeq -p_{a'\perp} \\
\langle p_{b'} p_b \rangle &\simeq -|p_{b'\perp}| \\
\langle p_a p_i \rangle &\simeq \langle p_{a'} p_i \rangle = -\sqrt{\frac{p_{a'}^+}{p_i^+}}\, p_{i\perp}
\end{aligned}
\tag{74}
$$

$$
\begin{aligned}
\langle p_i p_b \rangle &\simeq -\sqrt{\frac{p_i^+}{p_{b'}^+}} |p_{b'\perp}| \\
\langle p_i p_{b'} \rangle &\simeq -\sqrt{\frac{p_i^+}{p_{b'}^+}} p_{b'\perp} \\
\langle p_i p_j \rangle &\simeq -\sqrt{\frac{p_i^+}{p_j^+}} p_{j\perp} \qquad (y_i \gg y_j) \, .
\end{aligned}
$$

B Stringy Decomposition of the One-loop Amplitude

It is instructive to compute eq. (41) also from the string-inspired decomposition of a one-loop gluonic amplitude [34],

$$
m_{4:1} = N_c A_4^g + (4N_c - N_f)\, A_4^f + (N_c - N_f)\, A_4^s \, , \tag{75}
$$

with A_4^g the $N = 4$ supersymmetric multiplet, A_4^f the $N = 1$ chiral multiplet, and A_4^s the contribution of a complex scalar,

$$
A_4^x = c_\Gamma \, M_4^{tree} \, (V^x + F^x) \qquad\qquad x = g, f, s \, . \tag{76}
$$

The functions obtained from the $N = 4$ multiplet, V^g and F^g, are the same for any color ordering

$$
F^g = 0 \, , \qquad V^g = -\frac{2}{\epsilon^2} \left[\left(\frac{\mu^2}{-s_{12}} \right)^\epsilon + \left(\frac{\mu^2}{-s_{23}} \right)^\epsilon \right] + \ln^2 \left(\frac{-s_{12}}{-s_{23}} \right) + \pi^2 \, . \tag{77}
$$

The other functions depend on the color ordering. For orderings of type $m_{4:1}(-,-,+,+)$ we have

$$
\begin{aligned}
F^f = 0 \, , \quad V^f &= -\frac{1}{\epsilon} \left(\frac{\mu^2}{-s_{23}} \right)^\epsilon - 2 \, , \\
F^s = 0 \, , \quad V^s &= -\frac{V^f}{3} + \frac{2}{9} \, ;
\end{aligned} \tag{78}
$$

and for color orderings of type $m_{4:1}(-,+,-,+)$ we have

$$
\begin{aligned}
V^f &= -\frac{1}{2\epsilon} \left[\left(\frac{\mu^2}{-s_{12}} \right)^\epsilon + \left(\frac{\mu^2}{-s_{23}} \right)^\epsilon \right] - 2 \, , \\
V^s &= -\frac{V^f}{3} + \frac{2}{9} \, ,
\end{aligned} \tag{79}
$$

$$F^f = -\frac{s_{12}s_{23}}{2s_{13}^2}\left[\ln^2\left(\frac{-s_{12}}{-s_{23}}\right)+\pi^2\right]+\frac{s_{12}-s_{23}}{2s_{13}}\ln\left(\frac{-s_{12}}{-s_{23}}\right) ,$$
$$F^s = -\frac{s_{12}s_{23}}{s_{13}^2}\left[-\frac{s_{12}s_{23}}{s_{13}^2}\left(\ln^2\left(\frac{-s_{12}}{-s_{23}}\right)+\pi^2\right)+\frac{s_{12}-s_{23}}{s_{13}}\left(1+\frac{s_{13}^2}{6s_{12}s_{23}}\right)\ln\left(\frac{-s_{12}}{-s_{23}}\right)+1\right] .$$

Replacing the functions (76-79) into eq. (75) for the different color orderings and retaining only the leading powers in t/s, we obtain the unrenormalized amplitude (41) with $\delta_R = 0$. We note that to LL accuracy only the functions (77) obtained from the $N = 4$ multiplet contribute, while to NLL accuracy also the functions obtained from the $N = 1$ chiral multiplet and the complex scalar contribute.

C Absorptive parts of the 1-loop amplitudes in the high-energy limit

The absorptive parts of the 1-loop amplitudes in the high-energy limit can be obtained using the same techniques as in section 3. For the gluon-gluon amplitude we find

$$\begin{aligned}\mathrm{Absorp}\, M_4^{1-loop}(B-,A-,A'+,B'+) &= g^4\,\frac{s}{t}c_\Gamma\left(\frac{\mu^2}{-t}\right)^{\epsilon} N_c\,\frac{2}{\epsilon}\ln\frac{s}{u}\\ &\times\left\{i\,f^{aa'c}\,i\,f^{bb'c}-d^{aa'c}\,d^{bb'c}-\frac{2}{N_c}\left(\delta^{ab}\,\delta^{a'b'}+\delta^{ab'}\,\delta^{a'b}+2\delta^{aa'}\,\delta^{bb'}\right)\right\} ,\end{aligned} \tag{80}$$

where the d^{abc} are the symmetric $SU(3)$ structure constants and $\ln(s/u) \simeq -i\pi$ in the physical region. Both the $m_{4:1}$ subamplitudes (34, 35) and the $m_{4:3}$ subamplitudes (37, 38) contribute to this expression. Note that the term containing $f^{aa'c}f^{bb'c}$ agrees with the high-energy limit given in eq. (27).

For quark-quark scattering we find

$$\begin{aligned}\mathrm{Absorp}\, M_4^{1-loop}(B-,A-;A'+,B'+) &= g^4\,\frac{s}{t}c_\Gamma\left(\frac{\mu^2}{-t}\right)^{\epsilon}\frac{2}{\epsilon}\ln\frac{s}{u}\\ &\times\left\{2\,\lambda^c_{a'\bar a}\,\lambda^c_{b'\bar b}\left(N_c+\frac{1}{N_c}\right)-\delta_{a'\bar b}\,\delta_{b'\bar a}\left(\frac{N_c^2-1}{N_c}\right)\right\} .\end{aligned} \tag{81}$$

Both the $a_{4:1}$ and $a_{4:2}$ subamplitudes (58, 59) contribute to this expression. The absorptive amplitude for quark-antiquark scattering can be obtained by exchanging $s \leftrightarrow u$.

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