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The Infrared Behavior of QCD Cross Sections at Next-to-Next-to-Leading Order

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Abstract

In this talk we examine how one-loop soft and collinear splitting functions occur in the calculation of next-to-next-to-leading order (NNLO) corrections to production rates, and we present the one-loop gluon soft and splitting functions, computed to all orders in the dimensional regularization parameter ϵ. We apply the one-loop gluon soft function to the calculation of the next-to-leading logarithmic corrections to the Lipatov vertex to all orders in ϵ.

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The single most important parameter of perturbative QCD is the strong coupling constant, α_s, which has been determined in several ways [1]. Some of the most promising ones are due to hadron production in e^+e^- collisions; e.g., the hadronic branching ratio of the Z^0 or global event shape variables in $e^+e^- \to 3$ jets. The hadronic branching ratio R_Z is known in perturbative QCD to three loops; however, the usefulness of this observable in the determination of α_s is limited by the sensitivity of R_Z to other Standard Model parameters [2] (for an overview, see ref. [3]). On the contrary, $e^+e^- \to 3$ jets, which is known only to next-to-leading order (NLO) [4, 5], does not suffer from the above limitations. Thus a next-to-next-to-leading order (NNLO) calculation of this process could yield a significant reduction of the theoretical uncertainty in the determination of α_s.

In order to understand the general features of a calculation at NNLO, we begin by outlining how a higher-order calculation of a scattering process is performed. At leading order (LO) in α_s the cross section is obtained by squaring the tree amplitudes. If n particles are produced in the scattering, each of them will be resolved in the final state. Thus no singularities appear in the LO cross section. At LO the coupling α_s is evaluated with one-loop running, so that there is an implicit dependence on an arbitrary renormalization scale μ_R. In addition, if one or both of the scattering particles are strongly interacting, the cross section will factorize into the convolution of parton density functions (to be determined experimentally) and a hard partonic cross section, which is computed as an expansion in α_s. This procedure introduces into both the parton densities and the partonic cross section a dependence on a second arbitrary parameter, the factorization scale μ_F [6]. Typically, the dependence on μ_R and μ_F is maximal at LO.

The calculation of the cross section at next-to-leading order (NLO) in α_s is less straightforward. Two series of amplitudes are required in the squared matrix elements: *a*) tree and one-loop amplitudes for the production of n particles; *b*) tree amplitudes for the production of $n+1$ particles. The one-loop amplitudes typically have virtual ultraviolet and infrared singularities, which may be regularized using dimensional regularization. This involves analytically continuing the loop momenta into $D = 4-2\epsilon$ dimensions, so that the one-loop amplitude is now a function of ϵ. If this is expanded in ϵ, the ultraviolet singularities appear as single poles in ϵ, which can be removed by renormalizing the amplitude. This introduces an explicit dependence on the renormalization scale μ_R.

At NLO the structure of the infrared singularities has been extensively studied. Virtual infrared singularities appear as double poles in ϵ, when they are both soft and collinear, and single poles in ϵ, when they are either soft or collinear. Real infrared singularities occur in the phase-space integral over the $n+1$ final-state particles of the squared tree amplitudes, either when any gluon becomes soft or when any two massless particles become collinear, thus yielding single poles in ϵ. If one of the two collinear particles is soft, a double pole in ϵ arises. The singularities occur in a universal way, i.e. independent of the particular amplitude considered. Accordingly, soft singularities can be accounted for by universal tree soft functions [7, 8], and collinear singularities by universal tree splitting functions [9]. These have also been combined into a single function [10]. A detailed discussion of the infrared singularities at NLO for $e^+e^- \to$ jets may be found, for example, in ref. [11].

For processes with no strongly interacting scattering particles, all infrared divergences cancel when real and virtual contributions are put together to form the NLO coefficient in the expansion of the cross section [12]. Typically, the dependence on μ_R is reduced at NLO. For processes with strongly-interacting scattering particles, all infrared divergences cancel except for those associated with initial-state collinear singularities, which manifest themselves as single poles in ϵ; these singularities are factorized into the parton densities, thus reducing the dependence of the cross section on μ_F [6].

In order to compute a cross section at NNLO, three series of amplitudes are required: *a)* tree, one-loop, and two-loop amplitudes for the production of n particles; *b)* tree and one-loop amplitudes for the production of $n+1$ particles; *c)* tree amplitudes for the production of $n+2$ particles. For the case of NNLO $e^+e^- \to 3$ jets the five-parton final-state tree [13] amplitudes, as well as the four-parton final-state one-loop amplitudes exist in both helicity [14] and squared matrix-element forms [15]. However, as we discuss below, in order to be used in NNLO computations higher-order terms in ϵ must be included. For the required two-loop three-parton final-state amplitudes no computations exist, as yet. For single- and double-jet production at hadron colliders the six-parton tree [16, 17] amplitudes, as well as the five-parton one-loop amplitudes [18, 19, 20] exist in helicity matrix-element form, but no four-parton two-loop amplitude computations exist, as yet. Indeed, no two-loop amplitude computations exist for cases containing more than a single kinematic variable, except in the special cases of maximal supersymmetry [21].

In the calculation of a production rate at NNLO the structure of the infrared singularities is the following:

i) In the squared tree amplitudes, any two of the $n+2$ final-state particles can be unresolved. Accordingly the ensuing soft singularities, collinear singularities, and mixed collinear/soft singularities have been accounted for by double-soft functions [8], double-splitting functions and soft-splitting functions [22], respectively.

ii) In the interference term between a two-loop amplitude for the production of n particles and its tree-level counterpart, all the produced particles are resolved in the final state and no new singularities appear through the phase-space integration. Thus, the expansion of the two-loop amplitude in ϵ, which starts with a $1/\epsilon^4$ pole, can be truncated at $O(\epsilon^0)$. The universal structure of the coefficients of the $1/\epsilon^4$, $1/\epsilon^3$ and $1/\epsilon^2$ poles has been determined [23].

iii) In the interference term between a one-loop amplitude for the production of $n+1$ particles and its tree-level counterpart any one of the produced particles can be unresolved in the final state; hence, the phase-space integration gives at most an additional double pole in ϵ. Therefore, the expansion in ϵ of the interference term starts with a $1/\epsilon^4$ pole, from mixed virtual/real infrared singularities, and in order to evaluate it to $\mathcal{O}(\epsilon^0)$, the $(n+1)$-parton one-loop amplitude needs to be evaluated to $\mathcal{O}(\epsilon^2)$. (A similar need to evaluate one-loop amplitudes to higher orders in ϵ has been previously noted in NNLO deep inelastic scattering [24] and in the next-to-leading-logarithmic (NLL) corrections to the BFKL equation [25].)

iv) In the square of the one-loop amplitude for the production of n particles, the expansion in ϵ of the amplitude, which starts with a $1/\epsilon^2$ pole, must be known to $\mathcal{O}(\epsilon^2)$ in order to evaluate the squared amplitude to $\mathcal{O}(\epsilon^0)$.

Here we focus on the singularities in *iii)*, which require that the $(n+1)$-parton one-loop amplitudes be evaluated to $\mathcal{O}(\epsilon^2)$. For the case of NNLO corrections to $e^+e^- \to 3$ jets and to single- and double-jet production at hadron colliders, this would be a rather formidable task given the already non-trivial analytic structure of the one-loop $e^+e^- \to 4$ partons amplitudes [14, 15] and of the one-loop five-parton amplitudes [18, 19, 20], both presented through $\mathcal{O}(\epsilon^0)$ only. However, a simplification can be made if one uses the fact that the additional double poles in ϵ of the interference term arise from the infrared-divergent regions of the phase-space integration. This implies that the one-loop $(n+1)$-parton final-state amplitude needs be calculated to $\mathcal{O}(\epsilon^2)$ only in the regions where two partons become collinear or one parton becomes soft. Therefore, one can use this amplitude calculated to $\mathcal{O}(\epsilon^0)$ and then supplement it in the soft or collinear regions by appropriate $\mathcal{O}(\epsilon^2)$ terms. In these regions the amplitude factorizes into sums of products of n-parton final-state amplitudes multiplied by soft or collinear splitting functions. It is these soft or collinear splitting functions and the one-loop n-parton final-state amplitudes that must be evaluated to $\mathcal{O}(\epsilon^2)$. This is a much simpler task than evaluating the full one-loop $(n+1)$-parton final-state amplitudes beyond $\mathcal{O}(\epsilon^0)$.

Below, we provide the one-loop gluon splitting and soft functions to all orders in ϵ [29][‡]. A complete listing of the one-loop splitting and soft functions, including fermions, is given elsewhere [30]. Then we apply the framework outlined above to one of the effective vertices of the NLL corrections [31] to the BFKL equation [32], namely to the one-loop amplitude for three-parton production in multi-Regge kinematics [33, 25, 34], for which the produced partons are strongly ordered in rapidity. In NNLO and in NLL corrections to two-jet scattering, this amplitude appears in an interference term multiplied by its tree-level counterpart. Because of the rapidity ordering in the multi-Regge kinematics, the phase-space integration does not yield any collinear singularities; however, the gluon which is intermediate in rapidity can become soft. Accordingly the one-loop amplitude must be determined to $O(\epsilon^0)$ plus the contribution with the soft intermediate gluon evaluated to $O(\epsilon)$ [25, 34]. To determine the soft gluon contribution we use our all orders in ϵ determination of the soft functions together with previous all orders in ϵ determinations of the four-gluon amplitudes [35, 26, 36].

We first briefly review properties of n-gluon scattering amplitudes. The tree-level color decomposition is (see e.g. ref.[37] for details and normalizations)

$$M_n^{\rm tree}(1,2,\ldots n) = g^{(n-2)} \sum_{\sigma\in S_n/Z_n} {\rm Tr}\left(T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}\cdots T^{a_{\sigma(n)}}\right) m_n^{\rm tree}(\sigma(1),\sigma(2),\ldots,\sigma(n))\,, \qquad (1)$$

where S_n/Z_n is the set of all permutations, but with cyclic rotations removed. We have suppressed the dependence on the particle polarizations ε_i and momenta k_i, but label each leg with the index i. The T^{a_i} are fundamental representation matrices for the Yang-Mills gauge group $SU(N_c)$, normalized so that ${\rm Tr}(T^aT^b) = \delta^{ab}$. The behavior of color-ordered tree amplitudes as

[‡]The one-loop splitting functions through $\mathcal{O}(\epsilon^0)$ can be found in [26, 20], and the one-loop soft functions through $\mathcal{O}(\epsilon^0)$ may be extracted from the known four- [27] and five-parton [18, 28, 20] one-loop amplitudes.

the momenta of two color adjacent legs becomes collinear, is [37]

$$m_n^{\rm tree} \stackrel{a\|b}{\longrightarrow} \sum_{\lambda=\pm} {\rm Split}^{\rm tree}_{-\lambda}(a^{\lambda_a}, b^{\lambda_b})\, m_{n-1}^{\rm tree}(\ldots K^\lambda \ldots)\,, \tag{2}$$

where λ represents the helicity, $m_n^{\rm tree}$ are color-decomposed tree sub-amplitudes with a fixed ordering of legs and a and b are consecutive in the ordering, with $k_a = zK$ and $k_b = (1-z)K$. For the case of only gluons, the tree splitting functions splitting into a positive helicity gluon (with the convention that all particles are outgoing) is

$$\begin{aligned} {\rm Split}^{\rm tree}_{+}(a^+, b^+) &= 0\,, & {\rm Split}^{\rm tree}_{+}(a^-, b^-) &= \frac{-1}{\sqrt{z(1-z)}\,[a\,b]}\,,\\ {\rm Split}^{\rm tree}_{+}(a^-, b^+) &= \frac{z^2}{\sqrt{z(1-z)}\,\langle a\,b\rangle}\,, & {\rm Split}^{\rm tree}_{+}(a^+, b^-) &= \frac{(1-z)^2}{\sqrt{z(1-z)}\,\langle a\,b\rangle}\,, \end{aligned} \tag{3}$$

where the remaining ones may be obtained by parity. The spinor inner products [38, 17, 37] are $\langle i\,j\rangle = \langle i^-|j^+\rangle$ and $[i\,j] = \langle i^+|j^-\rangle$, where $|i^\pm\rangle$ are massless Weyl spinors of momentum k_i, labeled with the sign of the helicity. They are antisymmetric, with norm $|\,\langle i\,j\rangle\,| = |\,[i\,j]\,| = \sqrt{s_{ij}}$, where $s_{ij} = 2k_i\cdot k_j$.

The behavior of color-ordered tree amplitudes in the soft limit is very similar to the above. As the momentum k of an external leg becomes soft the color-ordered amplitudes become

$$m_n^{\rm tree}(..., a, k^\pm, b, ...)|_{k\to 0} = {\rm Soft}^{\rm tree}(a, k^\pm, b)\, m_{n-1}^{\rm tree}(..., a, b, ...)\,, \tag{4}$$

with the tree-level soft functions

$${\rm Soft}^{\rm tree}(a, k^+, b) = \frac{\langle a\,b\rangle}{\langle a\,k\rangle\,\langle k\,b\rangle}\,, \qquad {\rm Soft}^{\rm tree}(a, k^-, b) = \frac{-[a\,b]}{[a\,k][k\,b]}\,. \tag{5}$$

The factorization of the collinear (2) and of the soft (4) limits are similar. However, due to the locality of the collinear emission, the factorization property (2) extends to the full amplitude (1). Conversely, because of the non-locality of the soft emission and of the self-interactive nature of the gluon interaction, the factorization (4) is true only at the color-ordered amplitude level.

The color decomposition of one-loop multi-gluon amplitudes with adjoint states circulating in the loop is [39]

$$M_n^{\text{1-loop}}(1,2,\ldots n) = g^n \sum_{j=1}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;j}} {\rm Gr}_{n;j}(\sigma)\, m_{n;j}^{\text{1-loop}}(\sigma(1),\ldots,\sigma(n))\,, \tag{6}$$

where $\lfloor x\rfloor$ denotes the greatest integer less than or equal to x, ${\rm Gr}_{n;1}(1) \equiv N_c\,{\rm Tr}(T^{a_1}\cdots T^{a_n})$, ${\rm Gr}_{n;j}(1) = {\rm Tr}(T^{a_1}\cdots T^{a_{j-1}})\,{\rm Tr}(T^{a_j}\cdots T^{a_n})$ for $j>1$, and $S_{n;j}$ is the subset of permutations S_n that leaves the trace structure ${\rm Gr}_{n;j}$ invariant, and where $m_{n;j}^{\text{1-loop}}$ are color-decomposed one-loop sub-amplitudes. It turns out that at one-loop the $m_{n;j>1}$ can be expressed in terms of $m_{n;1}^{\text{1-loop}}$ [40], so we need only discuss this case here. The amplitudes with fundamental fermions in the loop contain only the $m_{n;1}^{\text{1-loop}}$ color structures and are scaled by a relative factor of $1/N_c$.

The behavior of color-ordered one-loop amplitudes as the momenta of two color adjacent legs becomes collinear, is [26, 20]

$$m_{n;1}^{\text{1-loop}} \stackrel{a\|b}{\longrightarrow} \sum_{\lambda=\pm}\Big\{\text{Split}_{-\lambda}^{\text{tree}}(a^{\lambda_a},b^{\lambda_b})\, m_{n-1;1}^{\text{1-loop}}(\ldots K^{\lambda}\ldots) + \text{Split}_{-\lambda}^{\text{1-loop}}(a^{\lambda_a},b^{\lambda_b})\, m_{n-1}^{\text{tree}}(\ldots K^{\lambda}\ldots)\Big\}\,. \tag{7}$$

The one-loop splitting functions are,

$$\begin{aligned}
\text{Split}_{+}^{\text{1-loop}}(a^-,b^-) &= (G^f+G^n)\,\text{Split}_{+}^{\text{tree}}(a^-,b^-)\,,\\
\text{Split}_{+}^{\text{1-loop}}(a^\pm,b^\mp) &= G^n\ \text{Split}_{+}^{\text{tree}}(a^\pm,b^\mp)\,,\\
\text{Split}_{+}^{\text{1-loop}}(a^+,b^+) &= -G^f\,\frac{1}{\sqrt{z(1-z)}}\,\frac{[a\,b]}{\langle a\,b\rangle^2}\,.
\end{aligned} \tag{8}$$

The function G^f arises from the 'factorizing' contributions and the function G^n arises from the 'non-factorizing' ones described in ref. [41] and are given through $\mathcal{O}(\epsilon^0)$ by [26, 20]

$$\begin{aligned}
G^f &= \frac{1}{48\pi^2}\Big(1-\frac{N_f}{N_c}\Big)z(1-z)+\mathcal{O}(\epsilon)\,,\\
G^n &= c_\Gamma\Big[-\frac{1}{\epsilon^2}\Big(\frac{\mu^2}{z(1-z)(-s_{ab})}\Big)^{\epsilon}+2\ln(z)\ln(1-z)-\frac{\pi^2}{6}\Big]+\mathcal{O}(\epsilon)\,,
\end{aligned} \tag{9}$$

with N_f the number of quark flavors and

$$c_\Gamma = \frac{1}{(4\pi)^{2-\epsilon}}\,\frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,. \tag{10}$$

As at tree-level, the remaining splitting functions can be obtained by parity. The explicit values were obtained by taking the limit of five-point amplitudes; the universality of these functions for an arbitrary number of legs was proven in ref. [41].

The functions (9) have been extended to all orders in ϵ in ref. [29]

$$\begin{aligned}
G^f &= \frac{2c_\Gamma}{(3-2\epsilon)(2-2\epsilon)(1-2\epsilon)}\Big(1-\epsilon\delta_R-\frac{N_f}{N_c}\Big)\Big(\frac{\mu^2}{-s_{ab}}\Big)^{\epsilon}z(1-z)\,,\\
G^n &= c_\Gamma\left(\frac{\mu^2}{-s_{ab}}\right)^{\epsilon}\frac{1}{\epsilon^2}\left[-\left(\frac{1-z}{z}\right)^{\epsilon}\frac{\pi\epsilon}{\sin(\pi\epsilon)}+2\sum_{k=1,3,5,\ldots}\epsilon^k\,\text{Li}_k\left(\frac{-z}{1-z}\right)\right]\,,
\end{aligned} \tag{11}$$

where the polylogarithms are defined as [42]

$$\left.\begin{aligned}
\text{Li}_1(z) &= -\ln(1-z)\\
\text{Li}_k(z) &= \int_0^z\frac{dt}{t}\,\text{Li}_{k-1}(t) \qquad (k=2,3,\ldots)
\end{aligned}\right\} = \sum_{n=1}^{\infty}\frac{z^n}{n^k}\,, \tag{12}$$

and the regularization scheme parameter is,

$$\delta_R = \begin{cases} 1 & \text{HV or CDR scheme,}\\ 0 & \text{FDH or DR scheme,}\end{cases} \tag{13}$$

where CDR denotes the conventional dimensional regularization scheme, HV the 't Hooft-Veltman scheme, DR the dimensional reduction scheme, and FDH the 'four-dimensional helicity scheme. (For further discussions on scheme choices see refs. [27, 43].)

The behavior of one-loop amplitudes in the soft limit, as the momentum k of an external leg becomes soft, is given by

$$\begin{aligned} & m_{n;1}^{\text{1-loop}}(...,a,k^\pm,b,...)|_{k\to 0} = \\ & \qquad \mathrm{Soft}^{\text{tree}}(a,k^\pm,b)\, m_{n-1;1}^{\text{1-loop}}(...,a,b,...) + \mathrm{Soft}^{\text{1-loop}}(a,k^\pm,b)\, m_{n-1}^{\text{tree}}(...,a,b,...)\,, \end{aligned} \tag{14}$$

where the one-loop gluon soft function may be extracted through $\mathcal{O}(\epsilon^0)$ from four- [27] and five-parton [18, 28, 20] one-loop amplitudes, and it is

$$\mathrm{Soft}^{\text{1-loop}}(a,k^\pm,b) = -\mathrm{Soft}^{\text{tree}}(a,k^\pm,b)\, c_\Gamma \left(\frac{\mu^2(-s_{ab})}{(-s_{ak})(-s_{kb})}\right)^\epsilon \left(\frac{1}{\epsilon^2}+\frac{\pi^2}{6}\right) + \mathcal{O}(\epsilon)\,. \tag{15}$$

Eq. (15) does not depend on N_f or δ_R. In ref. [29] we have extended it to all orders of ϵ, with the result,

$$\mathrm{Soft}^{\text{1-loop}}(a,k^\pm,b) = -\mathrm{Soft}^{\text{tree}}(a,k^\pm,b)\, c_\Gamma \frac{1}{\epsilon^2} \left(\frac{\mu^2(-s_{ab})}{(-s_{ak})(-s_{kb})}\right)^\epsilon \frac{\pi\epsilon}{\sin(\pi\epsilon)}\,. \tag{16}$$

We now apply the results for the soft function (16) to the case of three-gluon production in multi-Regge kinematics. To do so, we also need the four-gluon one-loop amplitude through $\mathcal{O}(\epsilon)$. In fact, this is known exactly to all orders of ϵ. In the high-energy limit, $s \gg t$, its dispersive part, which is all that contributes to the NLL BFKL kernel, is [29]

$$\begin{aligned} & \mathrm{Disp}\, M_4^{\text{1-loop}}(A^-,A'^+,B'^+,B^-) = M_4^{\text{tree}}(A^-,A'^+,B'^+,B^-)\, g^2\, c_\Gamma \left(\frac{\mu^2}{-t}\right)^\epsilon \frac{1}{\epsilon(1-2\epsilon)} \\ & \times \left\{ N_c \left[2(1-2\epsilon)\left(\psi(1+\epsilon) - 2\psi(-\epsilon) + \psi(1) + \ln\frac{s}{-t}\right) + \frac{1-\delta_R\epsilon}{3-2\epsilon} - 4 \right] + \frac{2(1-\epsilon)}{3-2\epsilon} N_f \right\}, \end{aligned} \tag{17}$$

where A, B and A', B' are respectively the incoming and outgoing gluons. The unrenormalized five-gluon one-loop amplitude in the multi-Regge kinematics, and in the soft limit for the intermediate gluon and to all orders in ϵ, is obtained by using eq. (14), with the four-gluon one-loop amplitude (17), and the dispersive part of the soft function (16), yielding [29]

$$\begin{aligned} & \mathrm{Disp}\, M_5^{\text{1-loop}}(A^-,A'^+,k^\pm,B'^+,B^-)|_{k\to 0} = g^2\, c_\Gamma\, M_5^{\text{tree}}(A^-,A'^+,k^\pm,B'^+,B^-)|_{k\to 0} \\ & \times \left[\left(\frac{\mu^2}{-t}\right)^\epsilon \left\{ N_c \left[-\frac{4}{\epsilon^2} + \frac{2}{\epsilon}\left(\psi(1+\epsilon) - 2\psi(1-\epsilon) + \psi(1) + \ln\frac{s}{-t}\right) \right.\right.\right. \\ & + \left.\left. \frac{1}{\epsilon(1-2\epsilon)}\left(\frac{1-\delta_R\epsilon}{3-2\epsilon} - 4\right)\right] + \frac{2(1-\epsilon)}{\epsilon(1-2\epsilon)(3-2\epsilon)} N_f \right\} \\ & - \left. N_c \left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \frac{1}{\epsilon^2}\left[1 + \epsilon\psi(1-\epsilon) - \epsilon\psi(1+\epsilon)\right] \right], \end{aligned} \tag{18}$$

which agrees through $\mathcal{O}(\epsilon^0)$ with the five-gluon one-loop amplitude, with strong rapidity ordering and in the soft limit for the intermediate gluon [18, 34]. Eq. (18) can than be matched to the full five-gluon one-loop amplitude, with strong rapidity ordering, computed through $\mathcal{O}(\epsilon^0)$. The result [34] agrees with the NLL corrections to the Lipatov vertex computed in ref. [25] in the CDR scheme, through $\mathcal{O}(\epsilon)$.

In conclusion, in this talk we have examined how one-loop soft and collinear splitting functions occur in the calculation of NNLO corrections to production rates, and we have presented the one-loop gluon soft and splitting functions, computed to all orders in ϵ. We have then applied the one-loop gluon soft function to the calculation of the NLL corrections to the Lipatov vertex to all orders in ϵ[29]. A systematic discussion of the soft and collinear splitting functions, including the case of external fermions, is presented elsewhere [30].

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