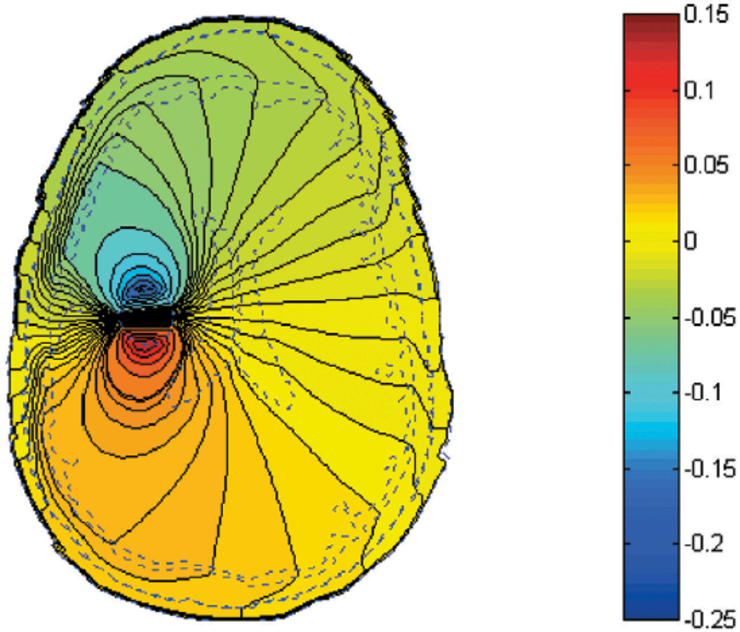
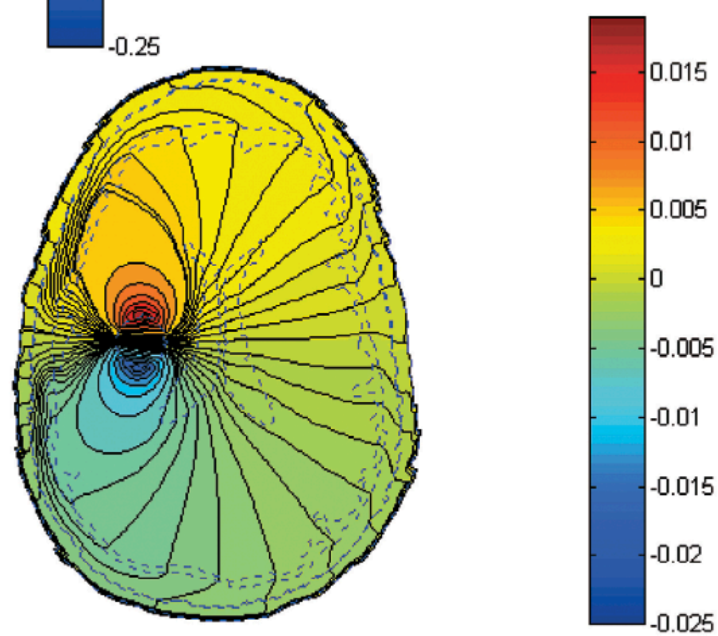


Electric Potential



Notes

- Correction set 1
  - Due Thursday evening at 10pm
  - Name: “Correction #1 RS”
  - Must answer all questions correctly to receive any credit
  - Gain 30% of the points you missed on the exam

Electric Potential Energy

- For a conservative force, the work is path-independent
- When an electrostatic force acts between two or more charges within a system, we can define an **electric potential energy**, U, in terms of the work done by the electric field, W_e, when the system changes its configuration from some initial configuration to some final configuration
  - The change in the electric potential energy is the negative of the work done by the electric field

    $$\Delta U = U_f - U_i = -W_e$$

    U_i is the initial electric potential energy

    U_f is the final electric potential energy

Electric Potential Energy

- We define the electric potential energy to be zero when all charges are infinitely far apart
- We can then write a simpler definition of the electric potential taking the initial potential energy to be zero,

$$\Delta U = U_{\mathrm{f}} - 0 = U = -W_{\mathrm{e},\infty}$$

- The negative sign on the work means
  - If E does positive work then $U < 0$
  - If E does negative work then $U > 0$

Electric Potential Difference ΔV

- The electric potential difference between an initial point i and final point f can be expressed in terms of the electric potential energy of q at each point

$$\Delta V = V_{\mathrm{f}} - V_{\mathrm{i}} = \frac{U_{\mathrm{f}}}{q} - \frac{U_{\mathrm{i}}}{q} = \frac{\Delta U}{q}$$

- Hence we can relate the change in electric potential to the work done by the electric field on the charge

$$\Delta V = -\frac{W_{\mathrm{e}}}{q}$$

- Taking the electric potential energy to be zero at infinity we have

$$V = -\frac{W_{\mathrm{e},\infty}}{q}$$

The Volt

- The commonly encountered unit joules/coulomb is called the **volt**, abbreviated V, after the Italian physicist Alessandro Volta (1745 - 1827)

$$1\text{ V} = \frac{1\text{ J}}{1\text{ C}}$$

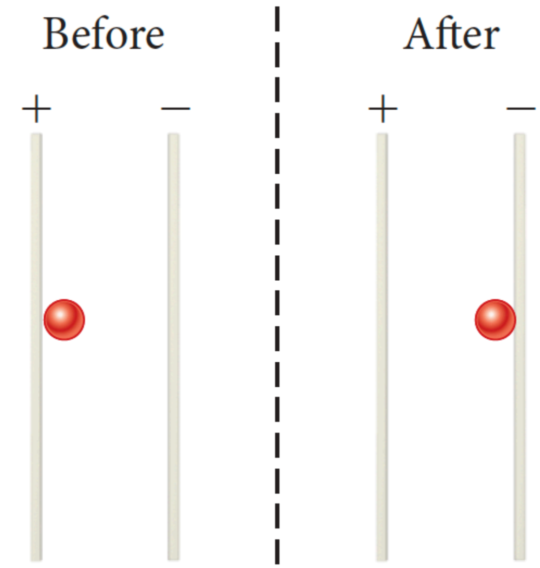
- With this definition of the volt, we can express the units of the electric field as

$$[E] = \frac{[F]}{[q]} = \frac{\text{N}}{\text{C}} = \frac{\text{J/m}}{\text{C}} = \frac{\text{V}}{\text{m}}$$

- For the remainder of our studies, we will use the unit V/m for the electric field

Energy Gain of a Proton

- A proton is placed between two parallel conducting plates in a vacuum
- The potential difference between the two plates is 450 V
- The proton is released from rest close to the positive plate



PROBLEM

- What is the kinetic energy of the proton when it reaches the negative plate?

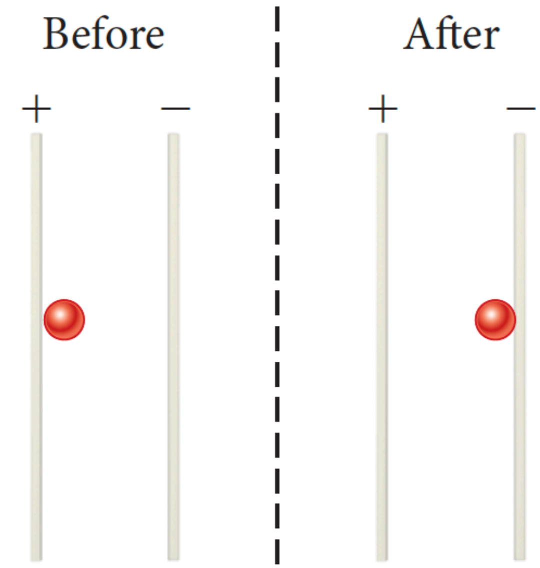
SOLUTION

- The electric potential difference between the plates is 450 V

Energy Gain of a Proton

- We can relate the electric potential difference across the plates to the change in electric potential energy

$$\Delta V = \frac{\Delta U}{q}$$



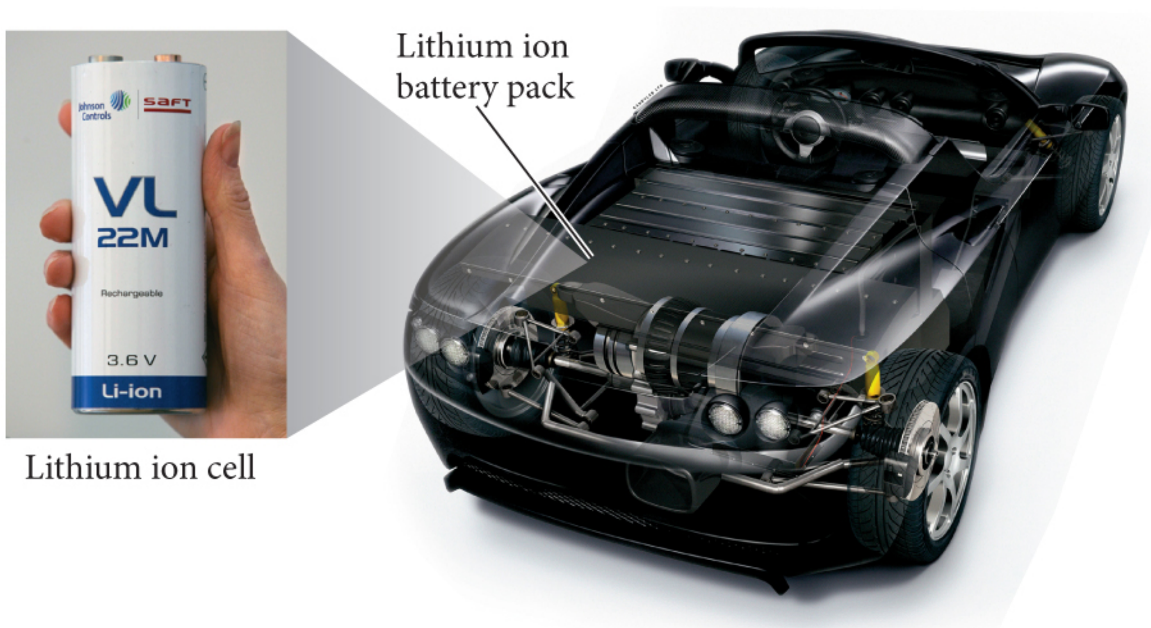
- All of the electric potential energy lost by the proton in crossing between the two plates is converted into kinetic energy
- Conservation of energy gives us

$$\Delta K + \Delta U = 0 \;\Rightarrow\; \Delta K = -\Delta U = -q\Delta V = K_f - 0 = K_f$$

$$K_f = -q\Delta V = -\left(1.602\cdot 10^{-19}\ \mathrm{C}\right)\left(-450\ \mathrm{V}\right) = 7.21\cdot 10^{-17}\ \mathrm{J}$$

Batteries

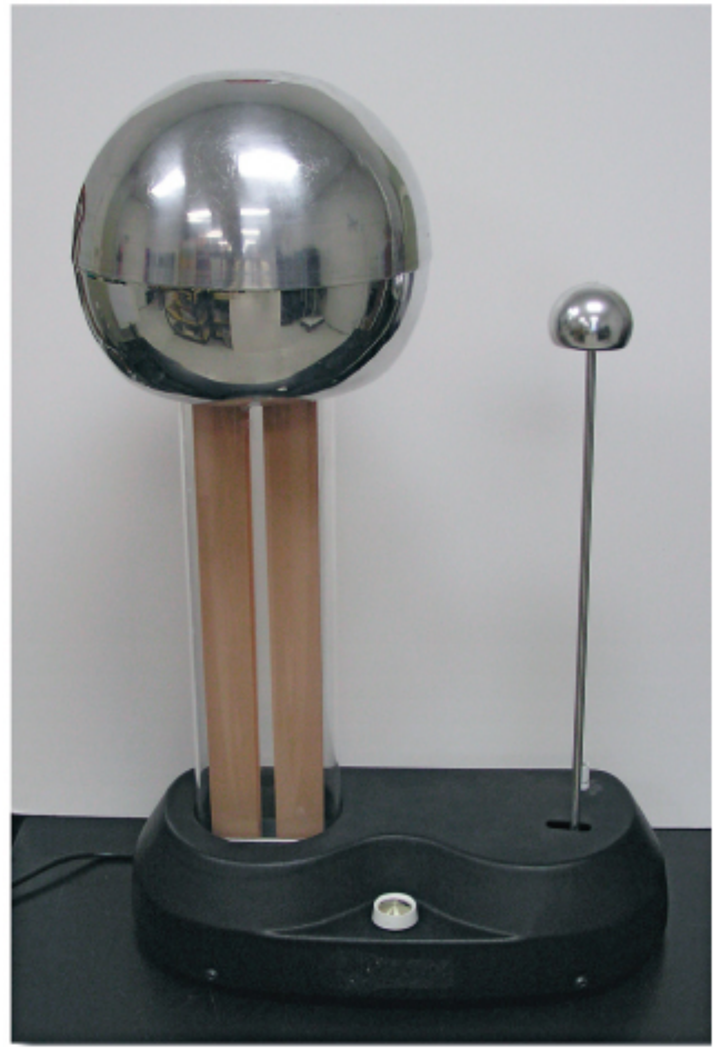
- A common method of creating an electric potential difference is a battery.
- A battery uses chemical processes to maintain a constant potential difference between its anode (–) and cathode (+).
- Lithium ion batteries are the focus of modern research.



(left): © Regis Duvignau/Reuters; (right): © Candy Lab Studios

Van de Graaff Generator

- A Van de Graaff generator is a device that creates high electric potential.
- The Van de Graaff generator was invented by Robert J. Van de Graaff, an American physicist (1901 - 1967).
- Van de Graaff generators can produce electric potentials up to many 10s of millions of volts.



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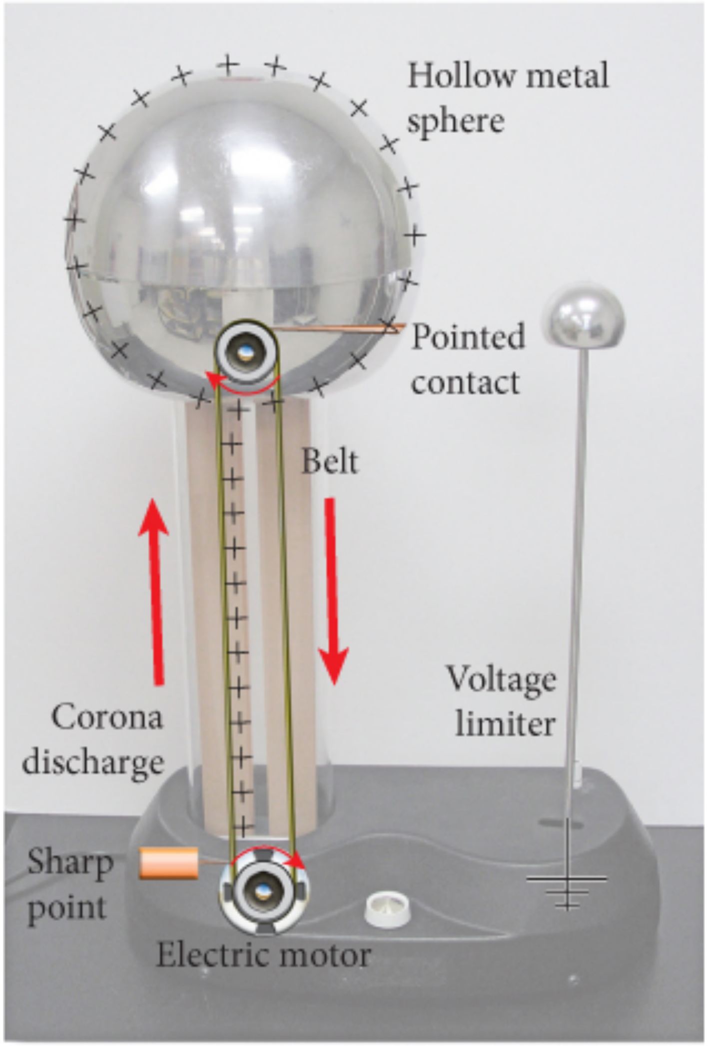
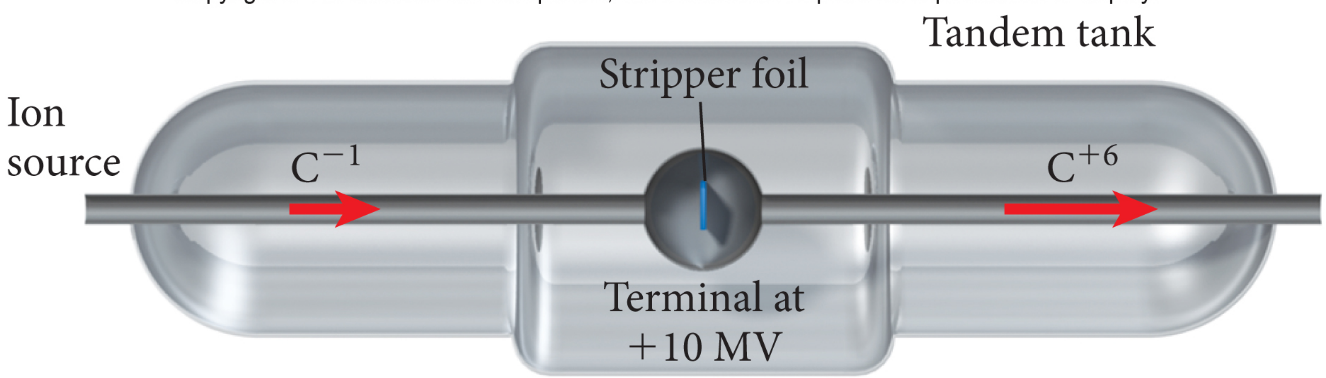
The Van de Graaff Generator

- The Van de Graaff generator works by applying a positive charge to a non-conducting moving belt using a corona discharge.
- The moving belt driven by an electric motor carries the charge up into a hollow metal sphere where the charge is taken from the belt by a pointed contact connected to the metal sphere.
- The charge that builds up on the metal sphere distributes itself uniformly around the outside of the sphere.



Tandem Van de Graaff Accelerator

- One use of a Van de Graaff generator is to accelerate particles for condensed matter and nuclear physics studies.
- A clever design is the tandem Van de Graaff accelerator:



- A large positive electric potential is created by a huge Van de Graaff generator.
- Negatively charged C ions get accelerated towards the +10 MV terminal (they gain kinetic energy).
- Electrons are stripped from the C and the now positively charged C ions are repelled by the positively charged terminal and gain more kinetic energy.

Tandem Van de Graaff Accelerator

- Suppose we have a tandem Van de Graaff accelerator that has a terminal voltage of 10 MV (10 million volts).
- We want to accelerate ^{12}C nuclei using this accelerator.

PROBLEM 1:

- What is the highest energy we can attain for carbon nuclei?

SOLUTION 1:

- There are two stages to the acceleration:
  - The carbon ions with a $-1e$ charge gain energy accelerating toward the terminal.
  - The stripped carbon ions with a $+6e$ charge gain energy accelerating away from the terminal.

Tandem Van de Graaff Accelerator

- The kinetic energy gained by each ion is:

$$\Delta K = |\Delta U| = |q_1 \Delta V| + |q_2 \Delta V| = K$$

$$K = e\Delta V + 6e\Delta V = 7e\Delta V$$

- Putting in our numerical values we get:

$$K = 7\left(1.602 \cdot 10^{-19}\ \mathrm{C}\right)\left(10 \cdot 10^{6}\ \mathrm{V}\right) = 1.12 \cdot 10^{-11}\ \mathrm{J}$$

- Physicists often use electron-volts instead of joules to express the kinetic energy of accelerated particles:

$$K = 7e\left(10 \cdot 10^{6}\ \mathrm{V}\right) = 7.0 \cdot 10^{7}\ \mathrm{eV} = 70\ \mathrm{MeV}$$

PROBLEM 2:

- What is the highest speed we can attain for carbon nuclei?

Tandem Van de Graaff Accelerator

SOLUTION 2:

- We can relate speed and kinetic energy:

$$K = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2K}{m}}$$

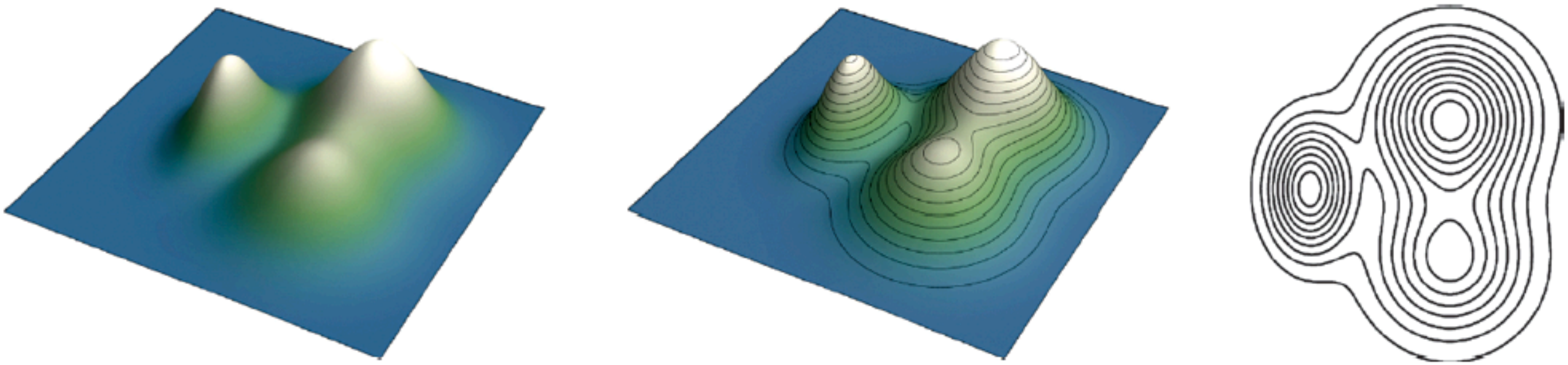
- The mass of a ^{12}C nucleus is $1.99 \cdot 10^{-26}$ kg so:

$$v = \sqrt{\frac{2\left(1.12 \cdot 10^{-11}\ \text{J}\right)}{1.99 \cdot 10^{-26}\ \text{kg}}} = 3.36 \cdot 10^{7}\ \text{m/s}$$

- This is 11% of the speed of light.

Equipotential Surfaces and Lines

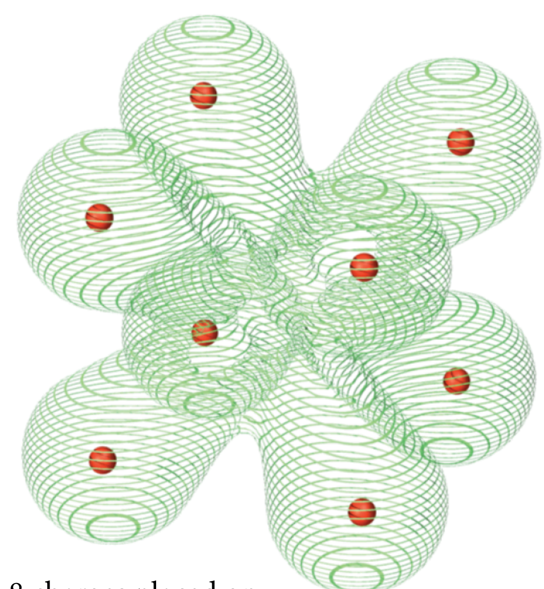
- Imagine you had to map out a ski resort with three peaks



- You could walk along the lines of equal elevation without going uphill or downhill
  - Lines of constant gravitational potential
- When an electric field is present, the electric potential has a value everywhere in space
- Points that have the same electric potential form an **equipotential surface**

Equipotential Surfaces and Lines

- Charged particles can move along an equipotential surface without having any work done on them by the electric field
- The surface of a conductor is an equipotential surface
- The electric field is zero everywhere inside the body of a conductor
- The entire volume of the conductor must be at the same potential
- Equipotential surfaces exist in three dimensions
- However, we usually take advantage of symmetries to represent the equipotential surfaces as **equipotential lines**



8 charges placed on the corners of a cube

General Considerations

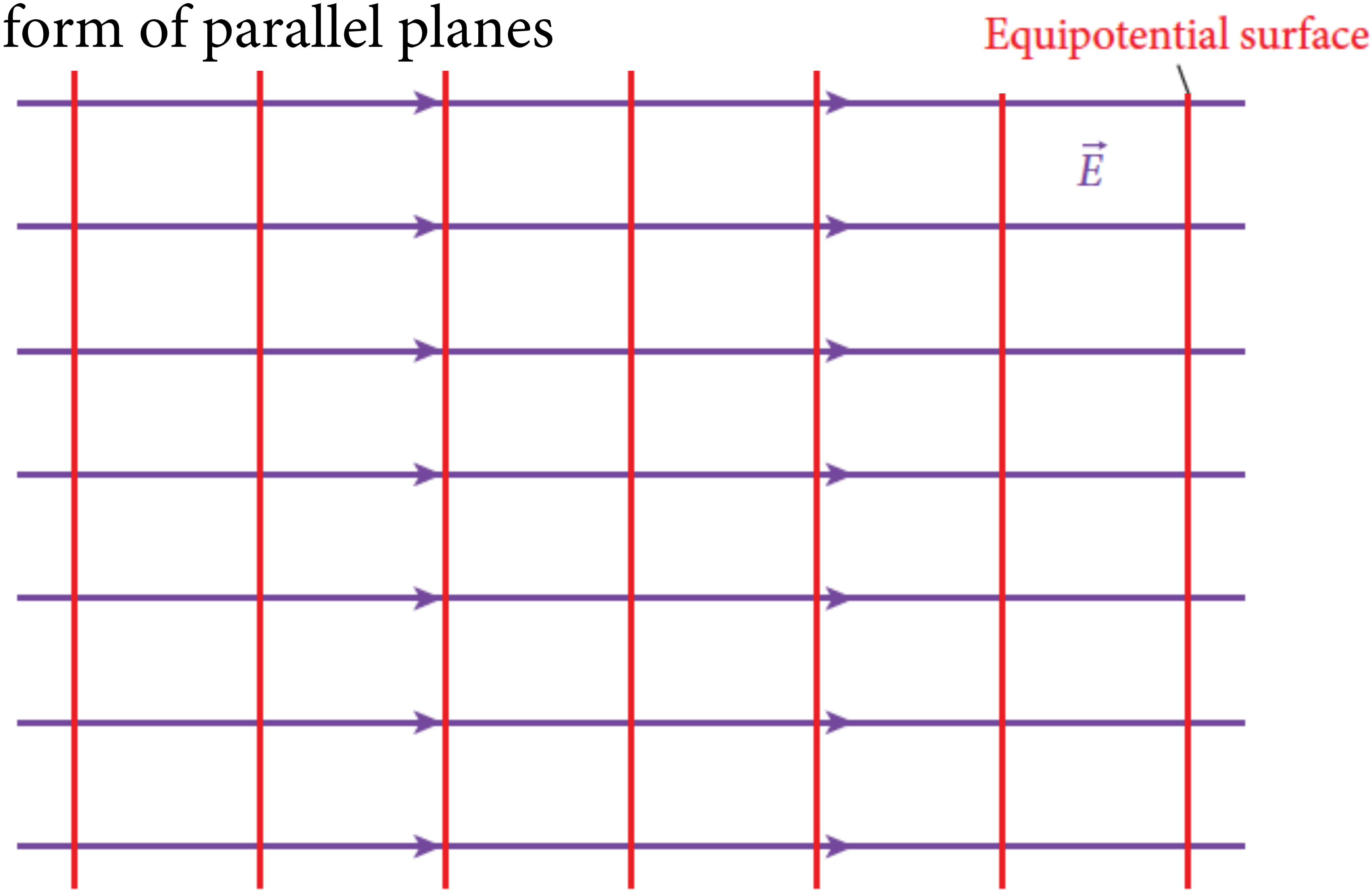
- If a charged particle moves perpendicular to electric field lines, no work is done

$$W = q\vec{E}\cdot\vec{d} = 0 \text{ if } \vec{d} \perp \vec{E}$$

- If the work done by the electric field is zero, then the electric potential must be constant
- Equipotential surfaces and lines must always be perpendicular to the electric field lines
- General observations:
    - The surface of any conductor forms an equipotential surface
    - Equipotential surface are always perpendicular to the electric field lines at any point in space

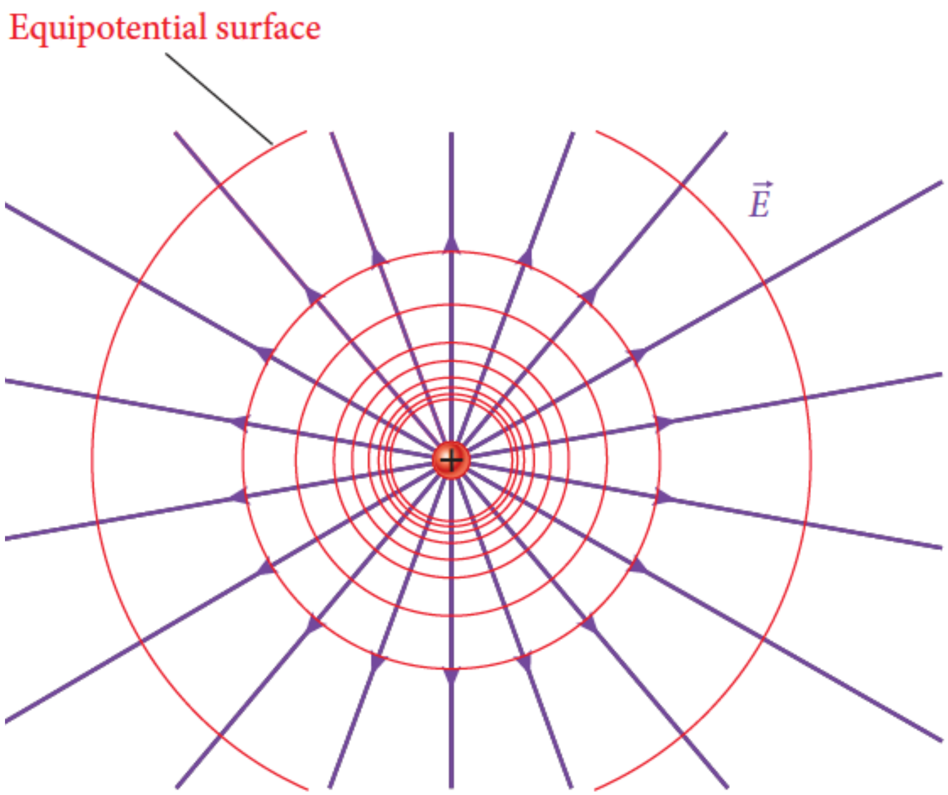
Constant Electric Field

- A constant electric field has straight, equally spaced, and parallel field lines, which produces equipotential surfaces in the form of parallel planes

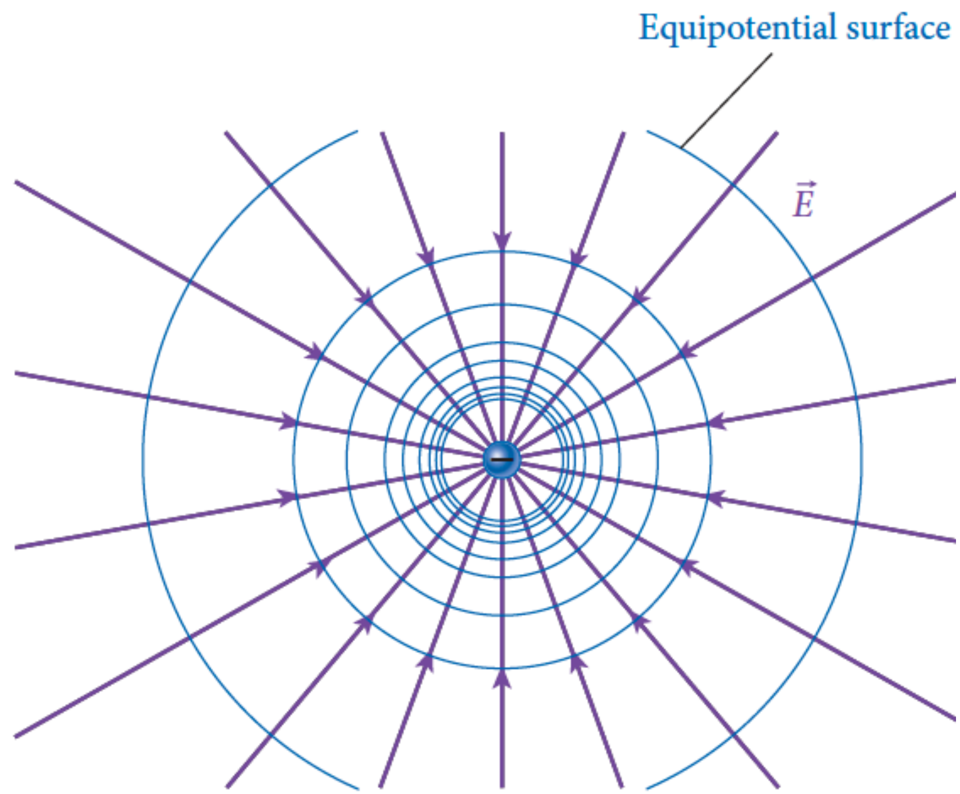


Single Point Charge

- The electric field lines for point charges are radial
- The equipotential surfaces are concentric spheres in 3D
- The equipotential lines take the form of concentric circles in 2D



Positive charge

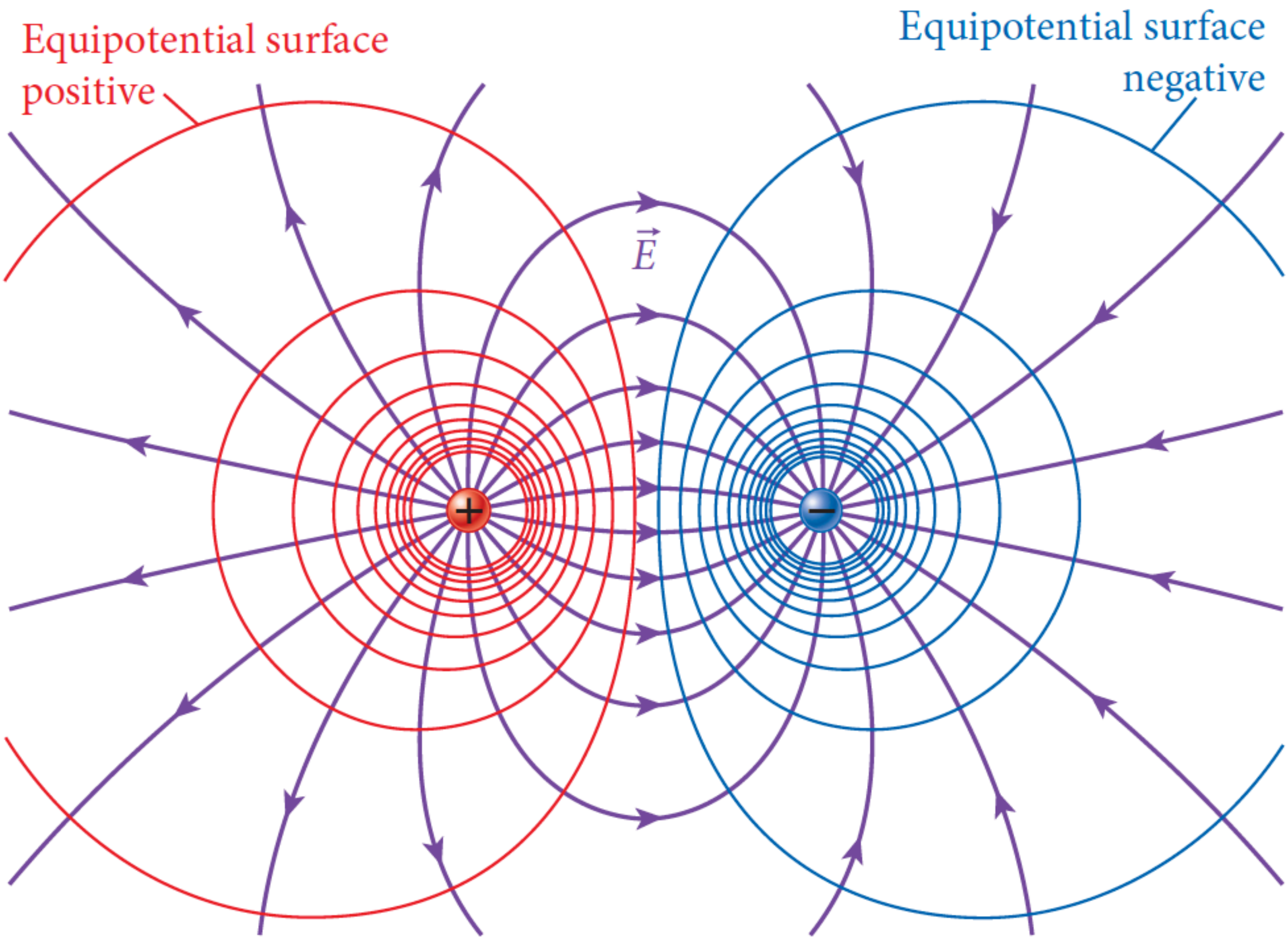


Negative charge

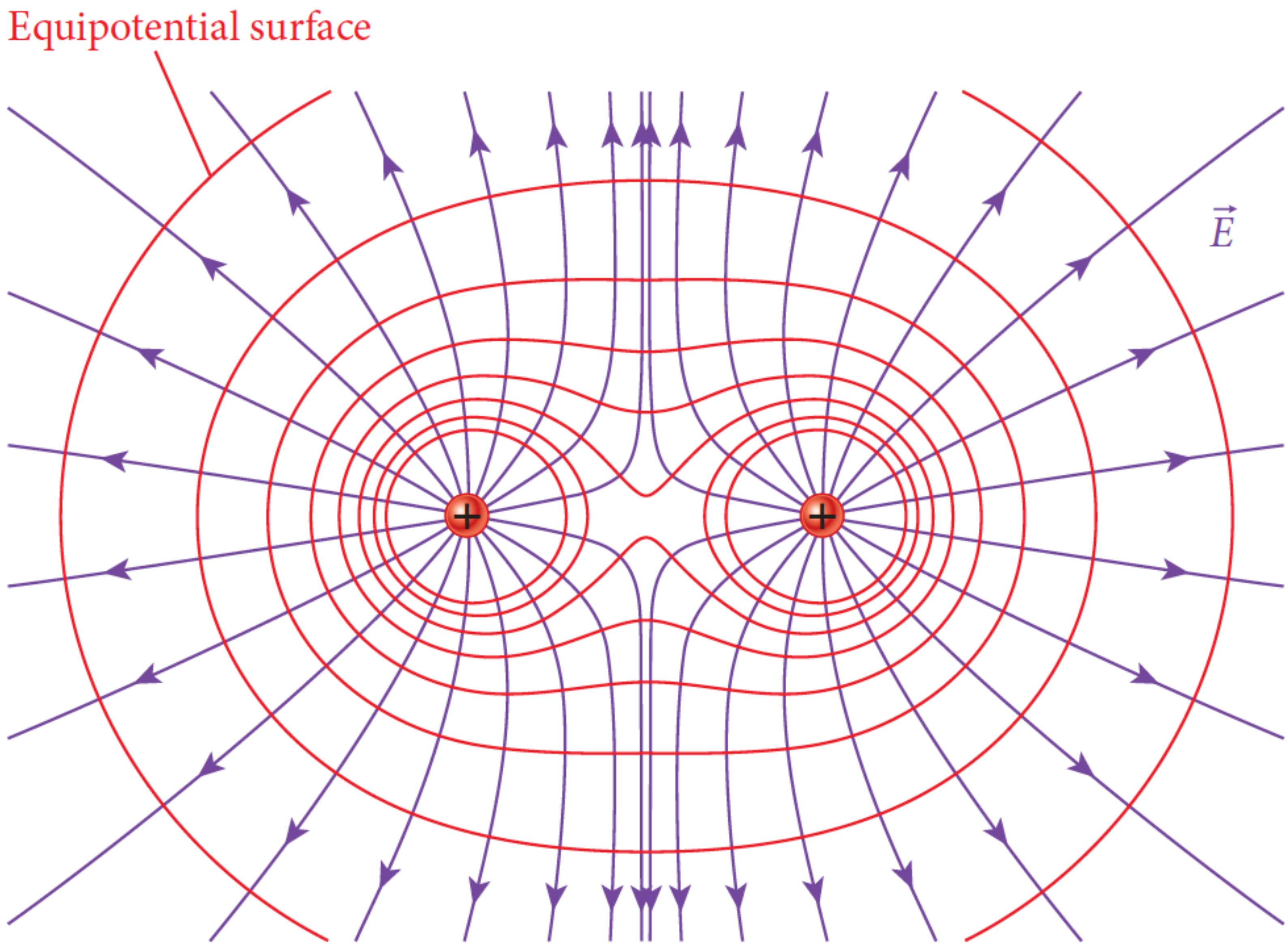
Two Oppositely Charged Point Charges

- The electric field lines from two oppositely charged point charges are a little more complicated
- The electric field lines originate on the positive charge and terminate on the negative charge
- The equipotential lines are always perpendicular to the electric field lines
- The red lines represent positive electric potential
- The blue lines represent negative electric potential
- Close to each charge, the equipotential lines resemble those from a point charge

Two Oppositely Charged Point Charges



Two Same-sign Point Charges



Electric Potential of Charge Distributions

- The electric potential is defined as the work required to place a unit charge at a point
- Work is a force acting over a distance
- The electric field is defined as the force acting on a unit charge at a point
- So the electric potential is related to the electric field
  - We can determine one given the other
- To determine the electric potential from the electric field we start with the work done on a charged particle by a force over a displacement

$$dW = \vec{F} \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$$

Electric Potential of Charge Distributions

- Now we integrate the work as the charge moves in the field

$$W = \int_{\mathrm{i}}^{\mathrm{f}} q\vec{E}\cdot d\vec{s} = q\int_{\mathrm{i}}^{\mathrm{f}} \vec{E}\cdot d\vec{s}$$

- Now we can relate the work done to the change in electric potential

$$\Delta V = V_{\mathrm{f}} - V_{\mathrm{i}} = -\frac{W_{\mathrm{e}}}{q} = -\int_{\mathrm{i}}^{\mathrm{f}} \vec{E}\cdot d\vec{s}$$

- Taking the standard convention, we can write the electric potential in terms of the electric field

$$V(\vec{r}) - V(\infty) = V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot d\vec{s}$$

- In other words, the electric potential is the integral of the electric field

Electric Potential for a Point Charge

- Let's calculate the electric potential due to a point charge
- The magnitude of the electric field is given by

$$E(r)=\frac{kq}{r^2}$$

- The direction of the electric field is radial from the charge
- Integrate along a radial line from ∞ to R

$$V(R)=-\int_{\infty}^{R}\vec{E}\cdot d\vec{s}=-\int_{\infty}^{R}\frac{kq}{r^2}dr=\left[\frac{kq}{r}\right]_{\infty}^{R}=\frac{kq}{R}$$

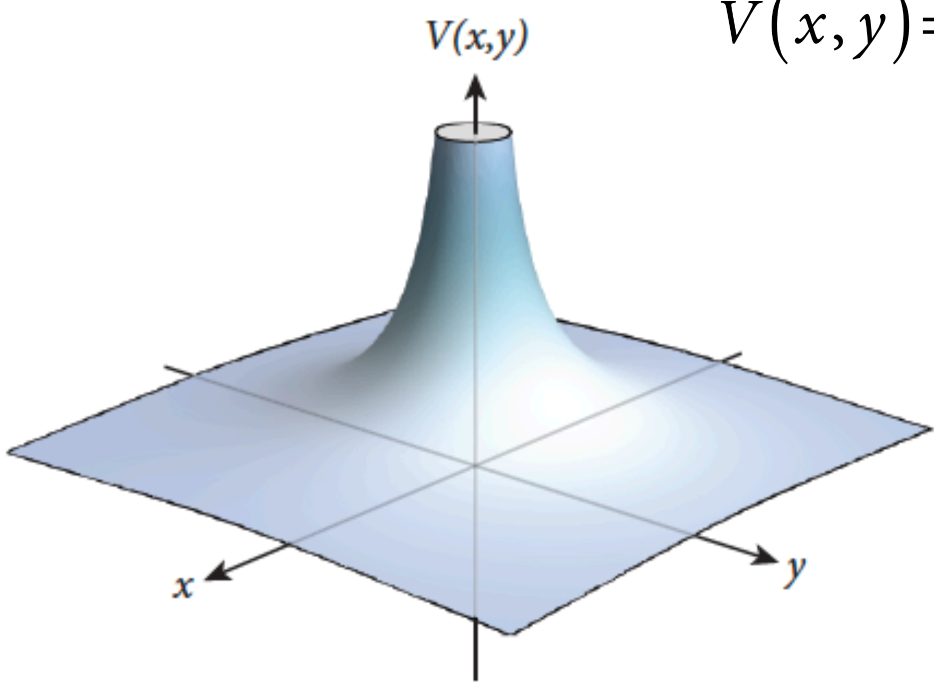
- So the electric potential due to a point charge is

$$V=\frac{kq}{R}$$

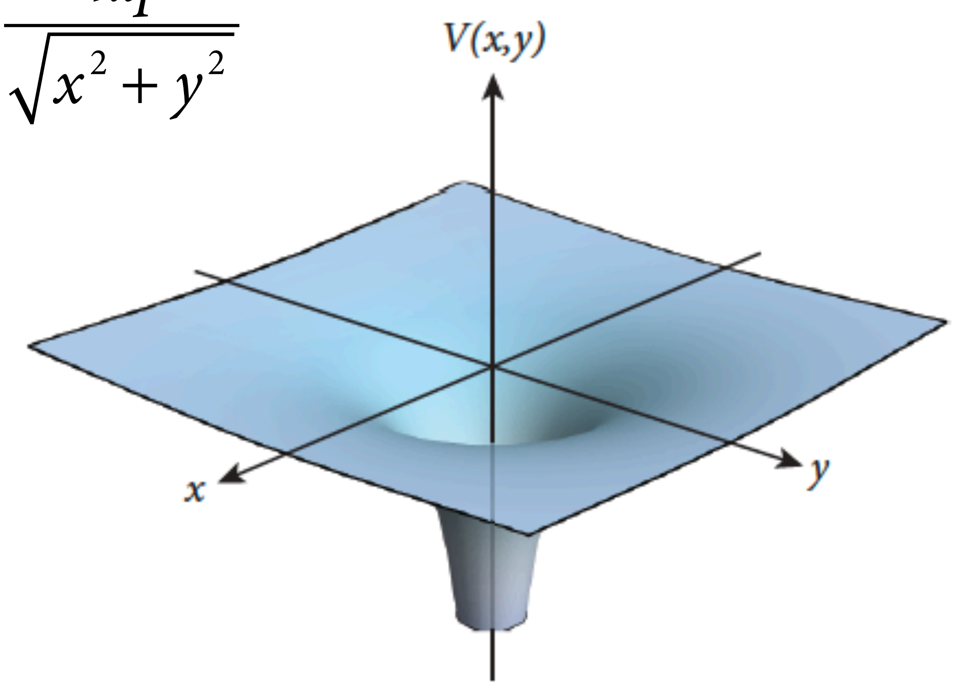
Electric Potential for a Point Charge

- A positive point charge produces a positive potential
- A negative point charge produces a negative potential
- We can calculate the electric potential for all points in an *xy*-plane

$$V(x,y)=\frac{kq}{\sqrt{x^2+y^2}}$$



$q > 0$



$q < 0$

Finding the Field from the Potential

- We can calculate the electric field from the electric potential starting with:

$$-qdV = q\vec{E} \bullet d\vec{s} \quad \Rightarrow \quad \vec{E} \bullet d\vec{s} = -dV$$

- If we look at the component of the electric field along the direction of *ds*, we can write the magnitude of the electric field as the partial derivative along the direction *s*:

$$E_s = -\frac{\partial V}{\partial s} \text{ in the direction of } d\vec{s}$$

- We can write the components of the electric field in terms of partial derivatives of the potential:

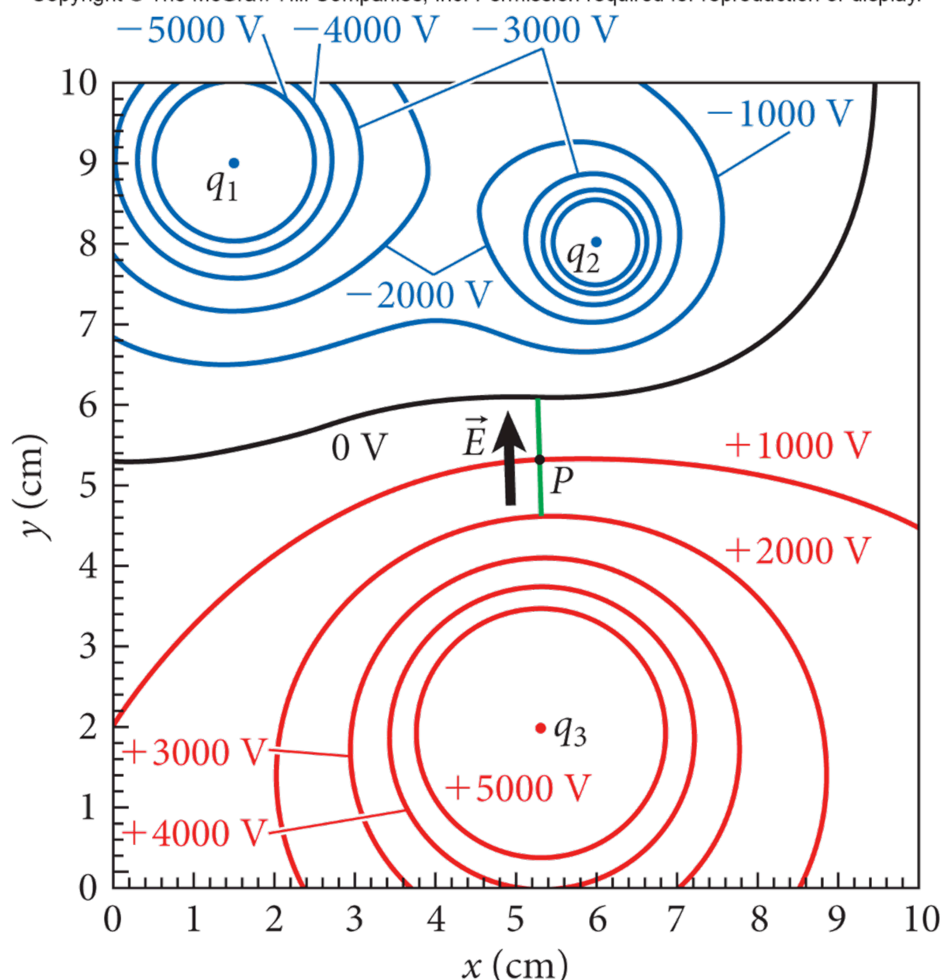
$$E_x = -\frac{\partial V}{\partial x}; \;\; E_y = -\frac{\partial V}{\partial y}; \;\; E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = -\vec{\nabla} V \equiv -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) \quad \vec{\nabla} \text{ is the gradient}$$

Graphical Extraction of the Electric Field

- Calculate the magnitude of the electric field at point *P*.
- To perform this task, we draw a line through point *P* perpendicular to the equipotential line reaching from the line of +2000 V to the line of 0 V.
- The length of this line is 1.5 cm.
- So the magnitude of the electric field can be approximated as

$$|E_s| = \left| -\frac{\Delta V}{\Delta s} \right|$$



Graphical Extraction of the Electric Field

- Putting in our numbers give us:

$$\left|E_s\right| = \left|\frac{(+2000\ \text{V}) - (0\ \text{V})}{1.5\ \text{cm}}\right|$$

$$\left|E_s\right| = 1.3 \cdot 10^5\ \text{V/m}$$

- The direction of the electric field points from the positive equipotential line to the negative potential line.