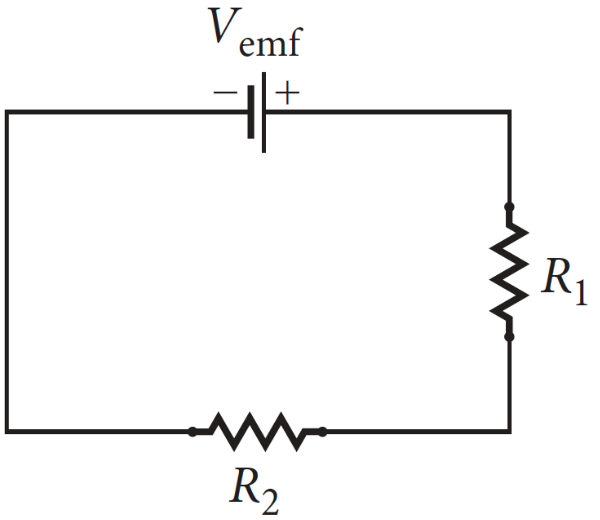
Direct Current Circuits

Resistors in series and in parallel

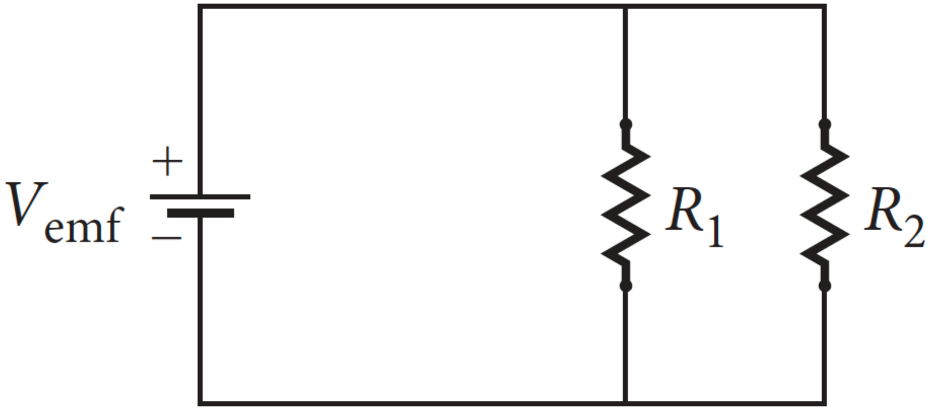
- Resistors in series:
  Replace two resistors in series by one resistor with equivalent resistance

$$R_{eq} = R_1 + R_2 \qquad R_{eq} = \sum_{i=1}^{n} R_i$$



- Resistors in parallel:
  Replace two resistors in parallel by one resistor with equivalent resistance

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \qquad \frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$



Energy and Power in Electric Circuits

- Energy in a circuit is given by

$$dU = dq\Delta V \qquad \text{or} \qquad dU = idt\Delta V$$

- Power is $P = dU/dt$, thus electric power is

$$P = i\Delta V = i^2 R = \frac{(\Delta V)^2}{R}$$

- i.e. power loss in a resistor is voltage drop across the resistor multiplied by current through the resistor
- The unit of power is the Watt (W)
- 1 W = 1 A V = 1 V^2/Ω = 1 $A^2\Omega$
- Thus energy is also in units of 1 J = 1 W s

Circuits

- So far, simple circuits with one emf device only.
- Capacitors wired in parallel

$$C_{\text{eq}} = \sum_{i=1}^{n} C_i$$

- Capacitors wired in series

$$\frac{1}{C_{\text{eq}}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

- Resistors wired in parallel

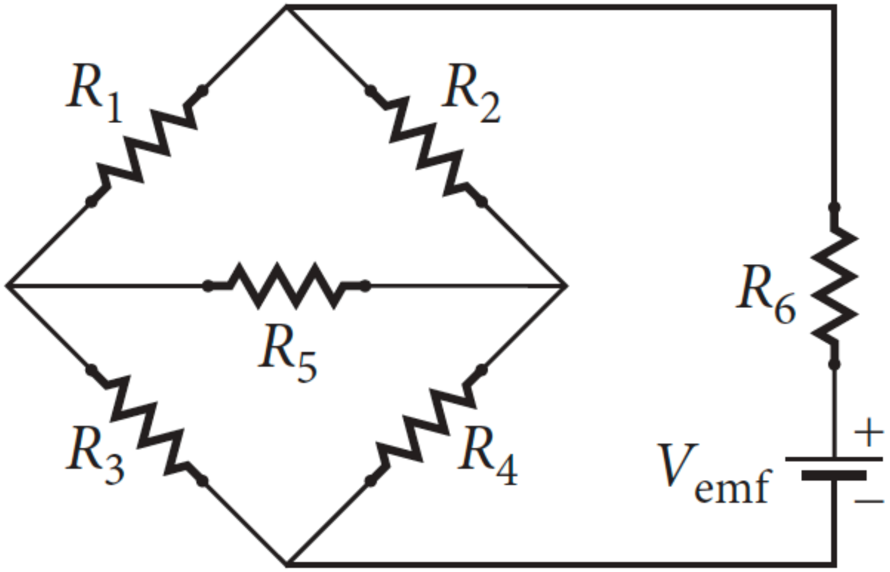
$$\frac{1}{R_{\text{eq}}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

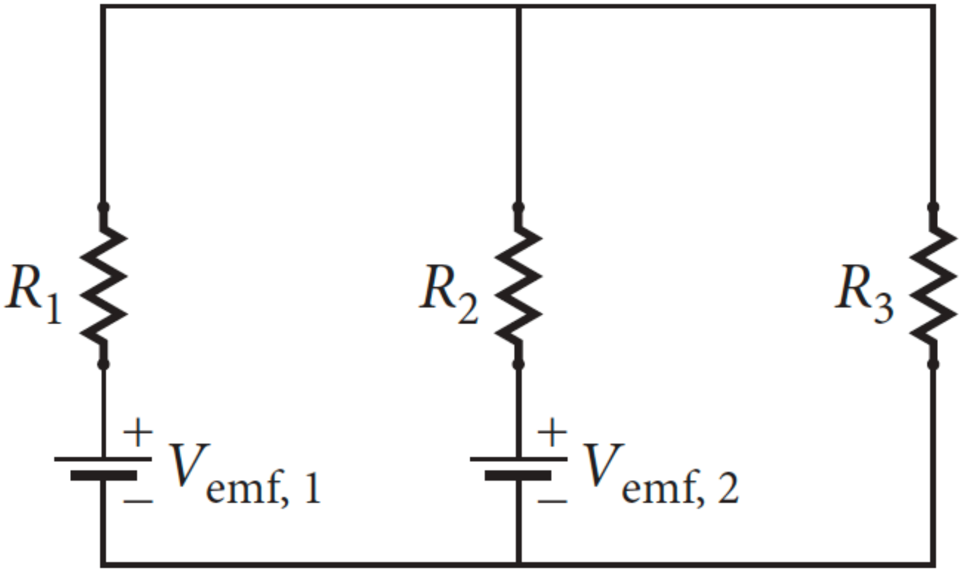
- Resistors wired in series

$$R_{\text{eq}} = \sum_{i=1}^{n} R_i$$

Complex Circuits

- Some circuits cannot be resolved into series or parallel systems of capacitors or resistors and contain more than one source of emf.
- To handle these types of circuits, we will apply **Kirchhoff's Rules**





Kirchhoff's Junction Rule

- A **junction** is a place in a circuit where three or more wires are connected to each other
- Each connection between two junctions in a circuit is called a **branch**
- A branch can contain any number of different circuit elements and the wires between them
- Each branch can have a current flowing, and this current is the same everywhere in the branch
- This leads to Kirchhoff's Junction Rule:

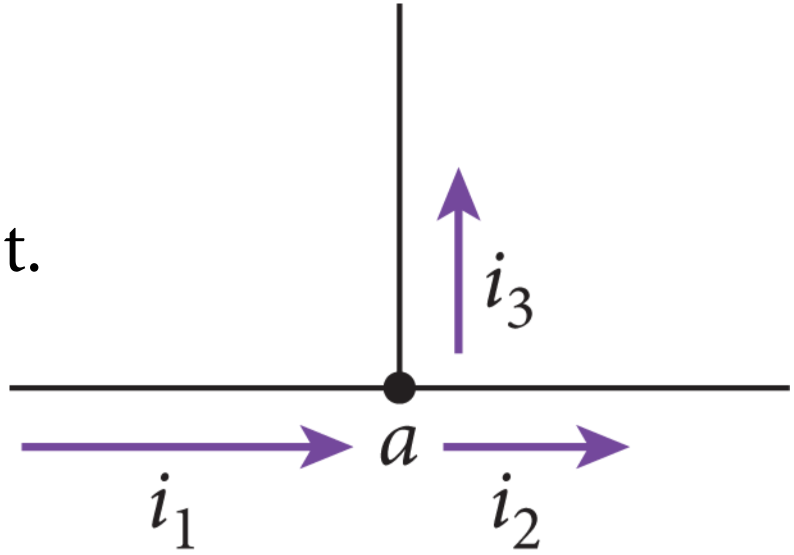
The sum of the currents entering a junction must equal the sum of the currents leaving the junction.

Kirchhoff's Junction Rule

- We assign a positive sign to currents entering the junction and a negative sign to currents exiting the junction: Kirchhoff's Junction Rule is then

$$\text{Junction: } \sum_{k=1}^{n} i_k = 0$$

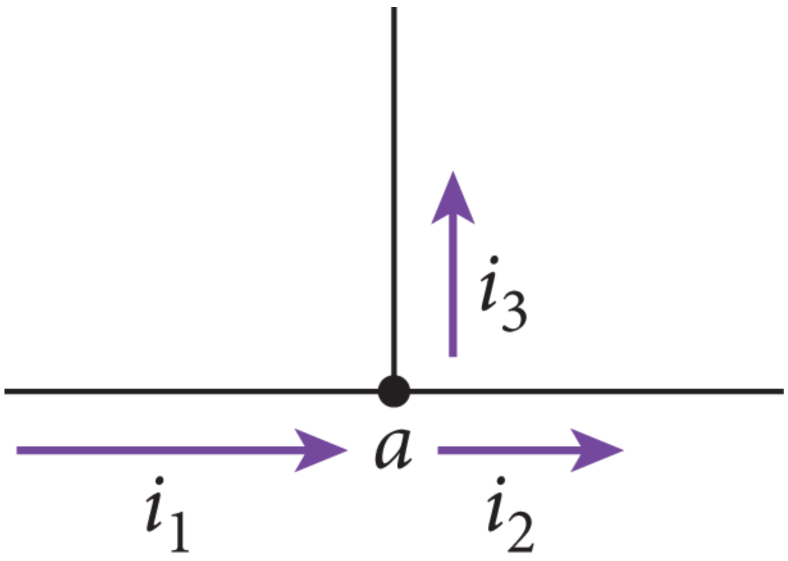
- Consider junction *a*, how do we know which currents enter and exit a junction? We don't.
- Simply assign a direction for each current
- If the assigned direction is wrong, we will get a negative number for that particular current.



Kirchhoff's Junction Rule

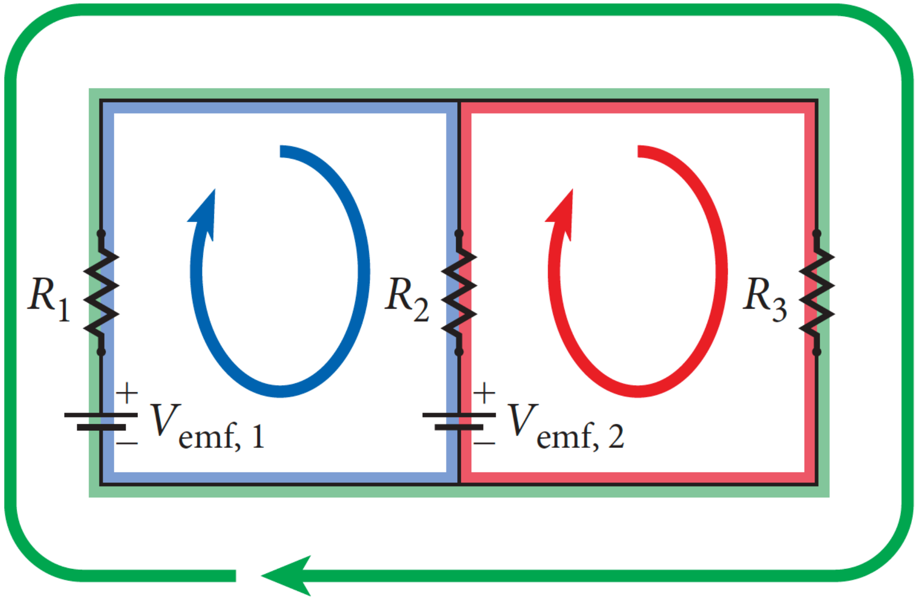
- Kirchhoff's Junction Rule is a direct consequence of the conservation of electric charge
- Junctions do not have the capability of storing charge
- Thus, charge conservation requires that all charges streaming into a junction also leave the junction
- At each junction in a multiloop circuit, the current flowing into must equal the current flowing out of the junction
- Example: Single junction, *a*, with a current, i_1, entering and two currents, i_2 and i_3, leaving the junction
- In this case,

$$\sum_{k=1}^{3} i_k = i_1 - i_2 - i_3 = 0 \quad \Rightarrow \quad i_1 = i_2 + i_3$$



Kirchhoff's Loop Rule

- A loop in a circuit is any set of connected wires and circuit elements forming a closed path
- If you follow a loop, eventually you will get to the same point from which you started
- For example, in the circuit diagram shown below, three possible loops can be identified



Kirchhoff's Loop Rule

- You can move through any loop in a circuit in either a clockwise or a counterclockwise direction
- The direction of the path taken around the loop is irrelevant as long as your choice is followed consistently all the way around the loop!
- Summing the potential differences from all circuit elements encountered along any given loop yields the total potential difference of the complete path along the loop.
- **Kirchhoff's Loop Rule** then states:

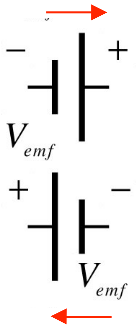
The potential difference around a complete circuit loop must sum to zero.

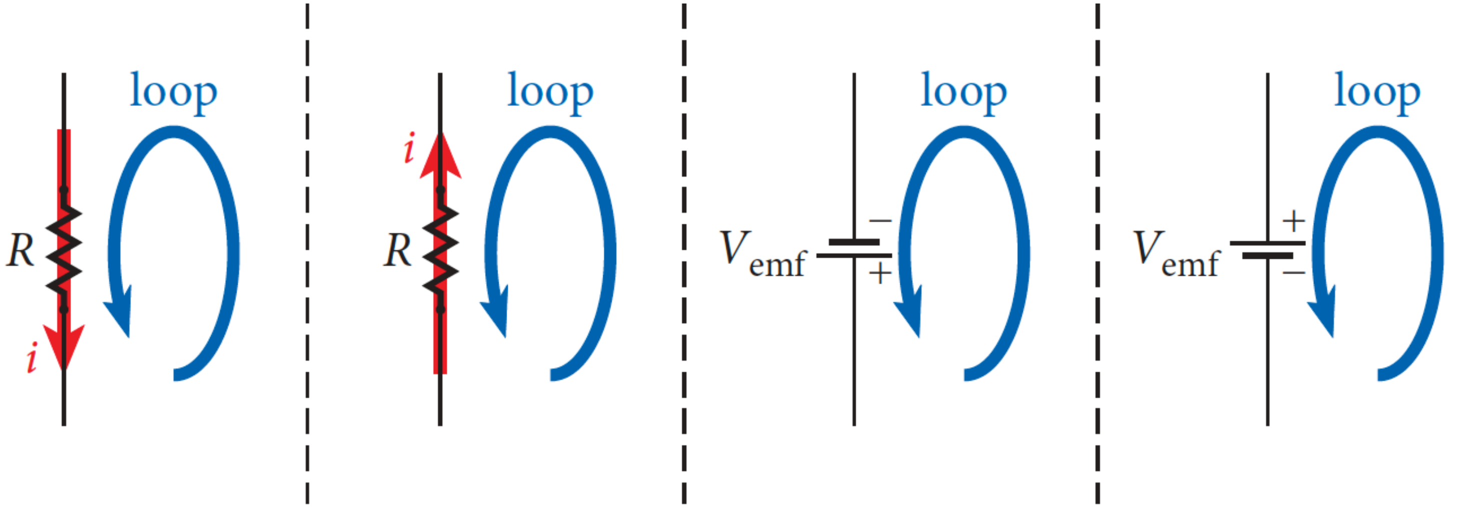
Kirchhoff's Loop Rule

- Kirchhoff's Loop Rule is a direct consequence of the concept of electric potential
- This means that the electric potential energy of a conduction electron at a point in the circuit has one specific value
- Kirchhoff's Loop Rule is simply a consequence of the law of conservation of energy
- Application of Kirchhoff's Loop Rule requires conventions for determining the potential drop across each element of the circuit
- This depends on the assumed direction of the current and the direction of the analysis

Sign Convention for Potential Changes

Element	Direction of Analysis	Potential Change	
R	Same as current	$-iR$	(a)
R	Opposite to current	$+iR$	(b)
V_{emf}	Same as emf	$+V_{emf}$	(c)
V_{emf}	Opposite to emf	$-V_{emf}$	(d)



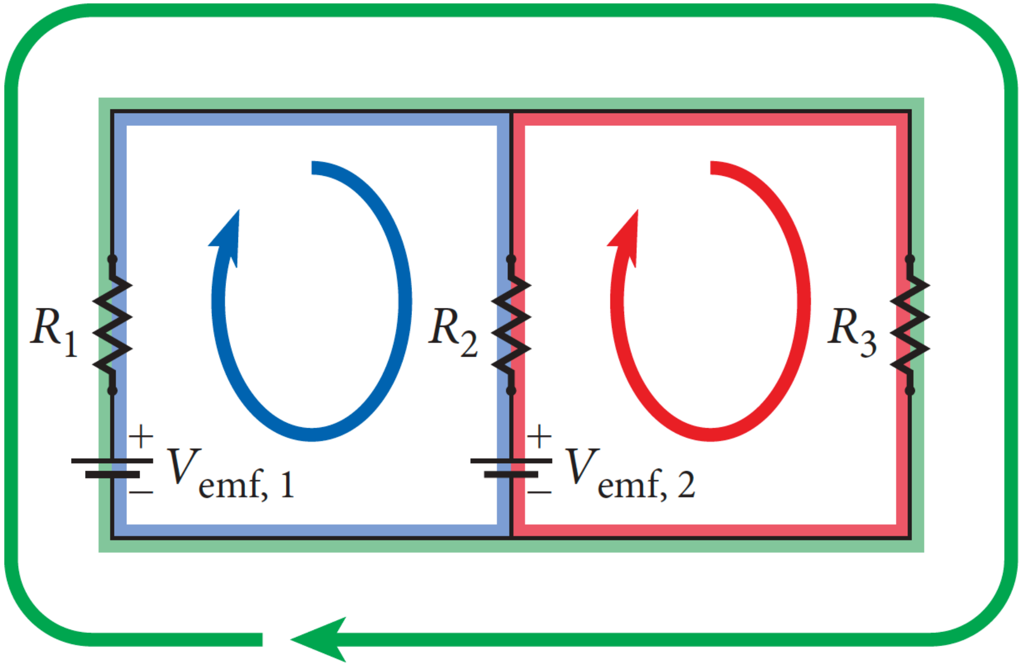


$\Delta V = -iR$ $\Delta V = +iR$ $\Delta V = +V_{emf}$ $\Delta V = -V_{emf}$

(a) **(b)** **(c)** **(d)**

Kirchhoff's Loop Rule

- **The potential difference around a complete circuit loop must sum to zero.**
- A loop in a circuit is any set of connected wires and circuit elements forming a closed path.
- The sign for voltage sources and resistors depends on the analysis direction and the current direction

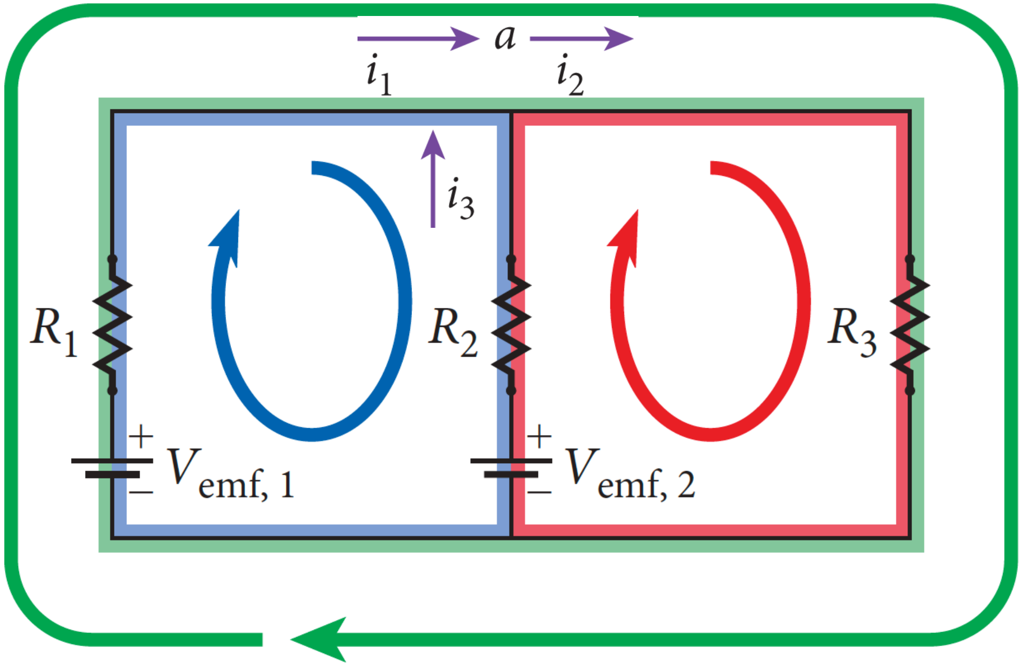


Kirchhoff's Loop Rule

- **The potential difference around a complete circuit loop must sum to zero.**
- A loop in a circuit is any set of connected wires and circuit elements forming a closed path.
- The sign for voltage sources and resistors depends on the analysis direction and the current direction

$$V_{emf,1} - i_1 R_1 + i_3 R_2 - V_{emf,2} = 0$$

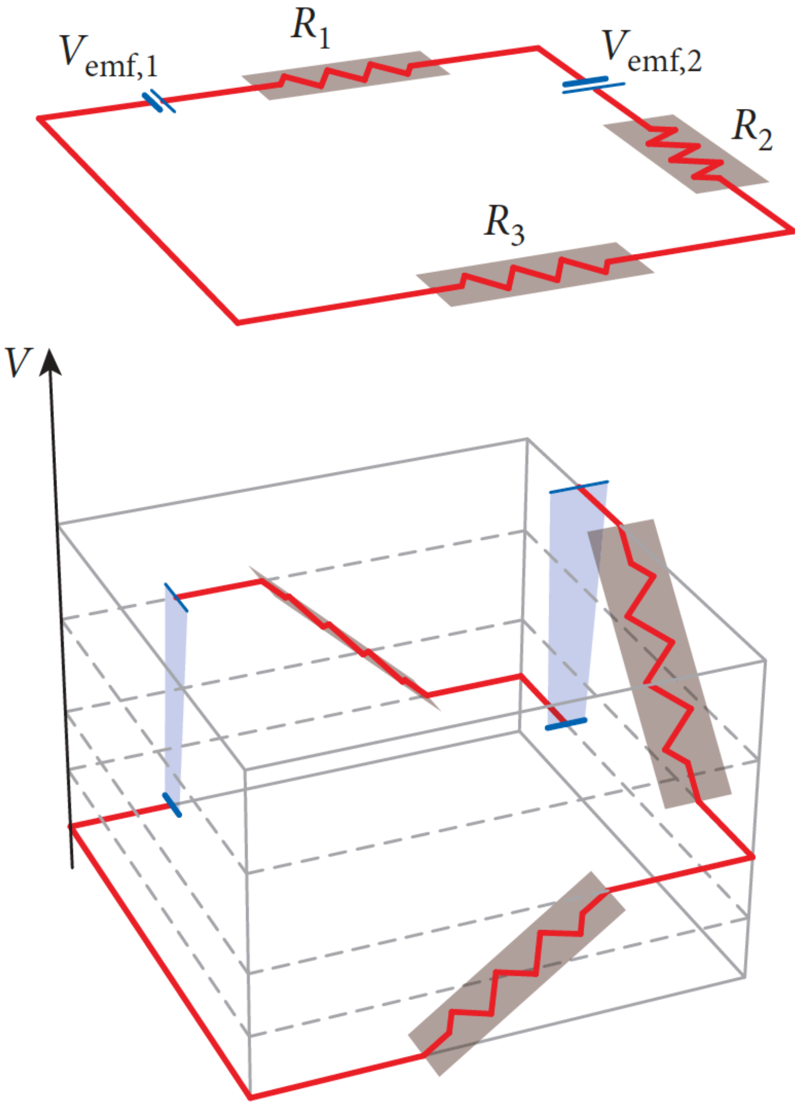
$$V_{emf,2} - i_3 R_2 - i_2 R_3 = 0$$



Kirchhoff’s Loop Rule

- Taking these conventions we can write Kirchhoff’s Loop Rule as

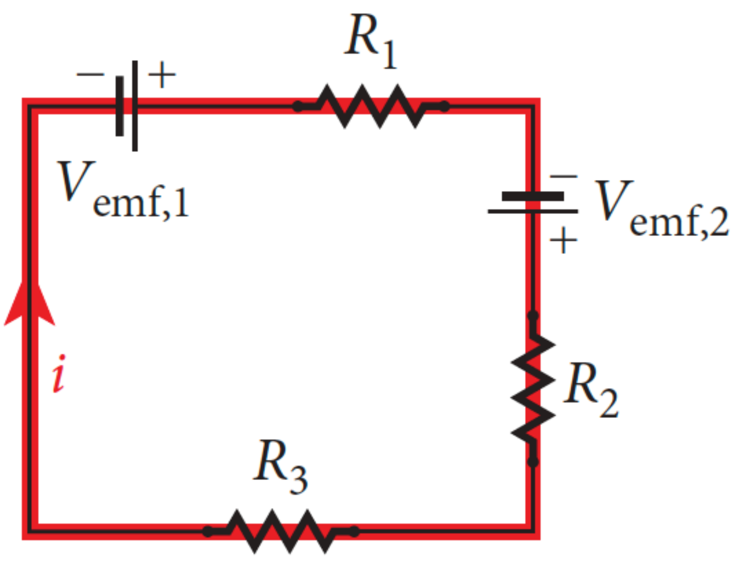
$$\text{Closed loop: } \sum_{j=1}^{m} V_{\mathrm{emf},j} - \sum_{k=1}^{n} i_k R_k = 0$$

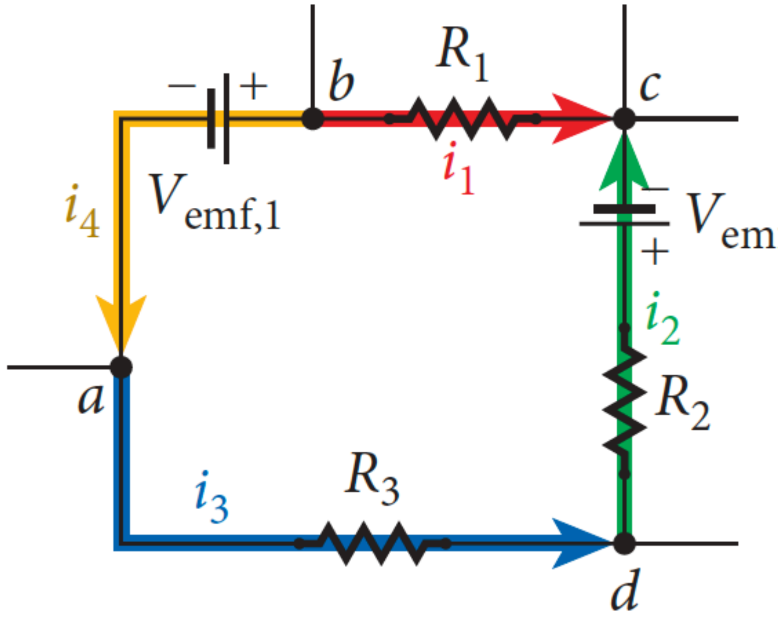


- Let’s look at a loop with two sources of emf and three resistors
- One complete turn around the loop always ends up at the same value of the potential as the starting point

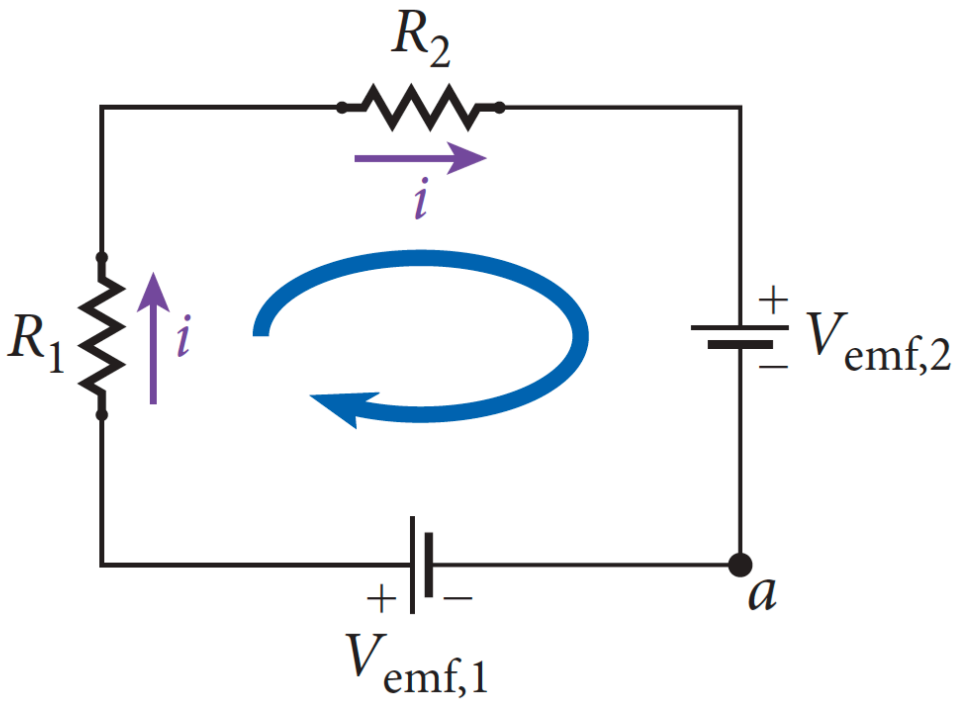
Kirchhoff's Loop Rule

- Let's look at the previous circuit in a standard representation as a single isolated loop
- The current is the same everywhere in the loop
- Now break the loop up into four branches with four junctions, each of which can have a different current flowing through it
- Kirchhoff's Loop Rule still holds

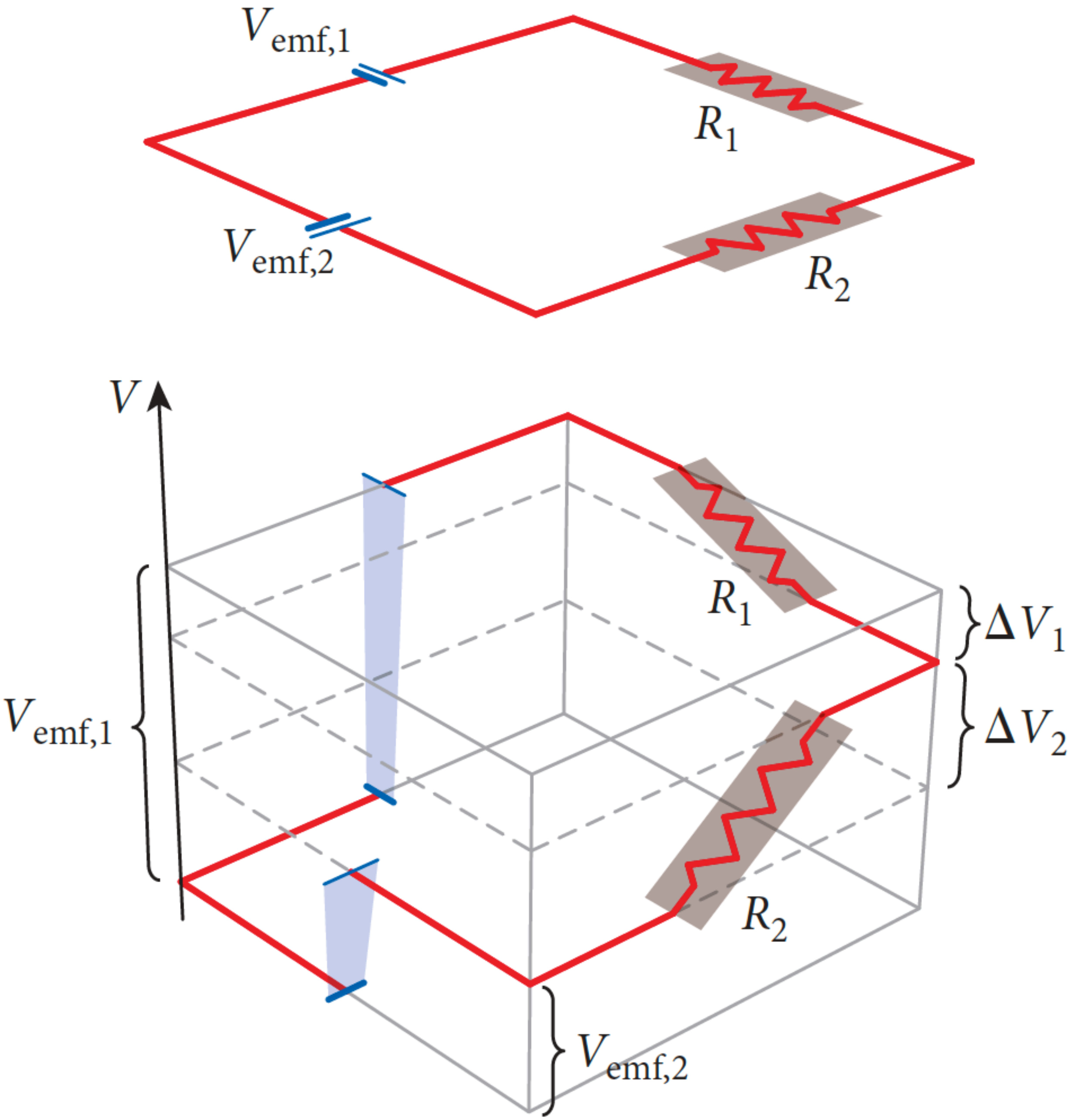




Single-Loop Circuits



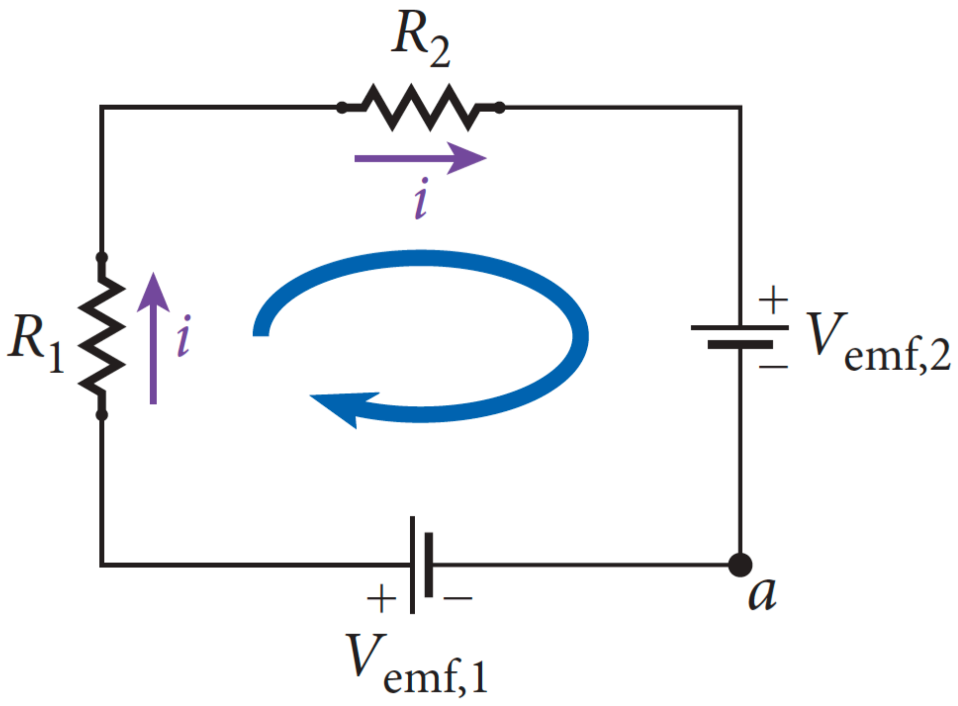
- Consider a circuit containing two sources of emf and two resistors connected in series in a single loop
- There are no junctions so entire circuit consists of one branch
- The current is the same everywhere in the loop as illustrated by the purple arrow


$V_{\text{emf},1}$
R_1
$V_{\text{emf},2}$
R_2
V
R_1
ΔV_1
$V_{\text{emf},1}$
ΔV_2
R_2
$V_{\text{emf},2}$

Analysis of Single-loop Circuits

- Choose a direction for the current
- We can determine if our assumption for the direction of the current is correct after the analysis is complete
- Resulting current positive
  - Current is flowing in the same direction as we had chosen
- Resulting current negative
  - Current is flowing in the direction opposite to what we had chosen
- We can choose the direction in which we analyze the circuit
  - Any direction we choose will give us the same information

Single-Loop Circuits



- Let's start at point *a* and move around the loop in a clockwise direction
- The first component is $V_{\text{emf},1}$, which produces a positive potential gain of $V_{\text{emf},1}$
- Next is resistor R_1 which produces a potential drop given by $-\Delta V_1 = iR_1$
- The next component is resistor R_2, which produces a potential drop given by $-\Delta V_2 = iR_2$
- Next we encounter a second source of emf, $V_{\text{emf},2}$, which produces a potential drop rather than a potential gain

Single-Loop Circuits

- We sum the potential changes around the loop

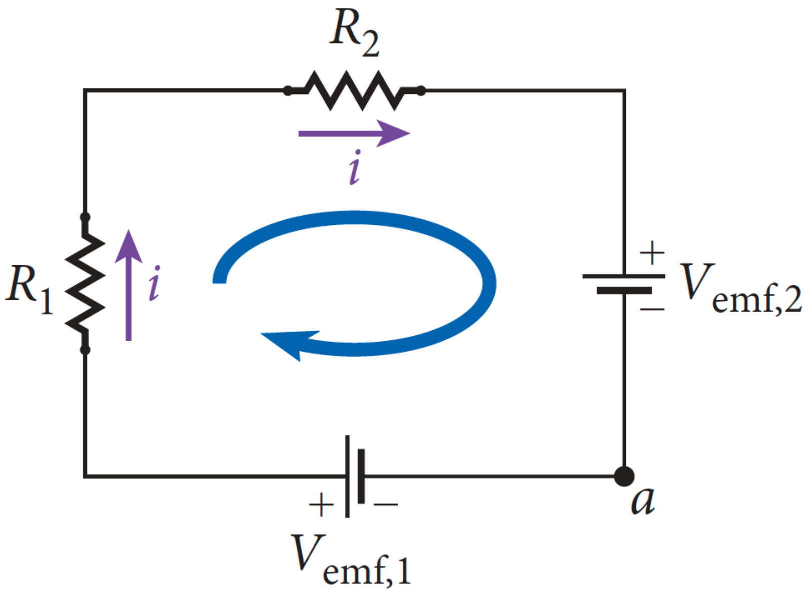
$$V_{\mathrm{emf},1} - \Delta V_1 - \Delta V_2 - V_{\mathrm{emf},2} = 0 \quad \Rightarrow$$

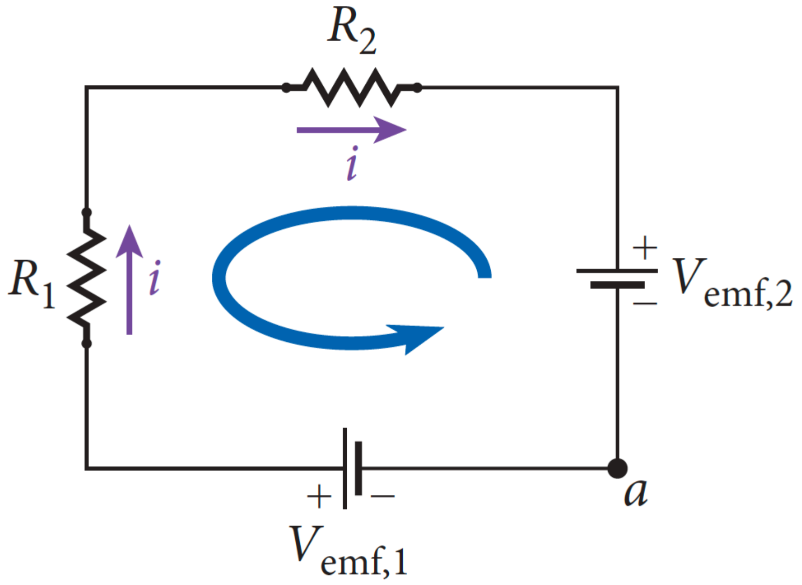
$$V_{\mathrm{emf},1} - iR_1 - iR_2 - V_{\mathrm{emf},2} = 0$$

- Now let's analyze the loop by going around the loop in the opposite direction

$$+V_{\mathrm{emf},2} + iR_2 + iR_1 - V_{\mathrm{emf},1} = 0$$

- Clockwise and counterclockwise loop directions give the same information



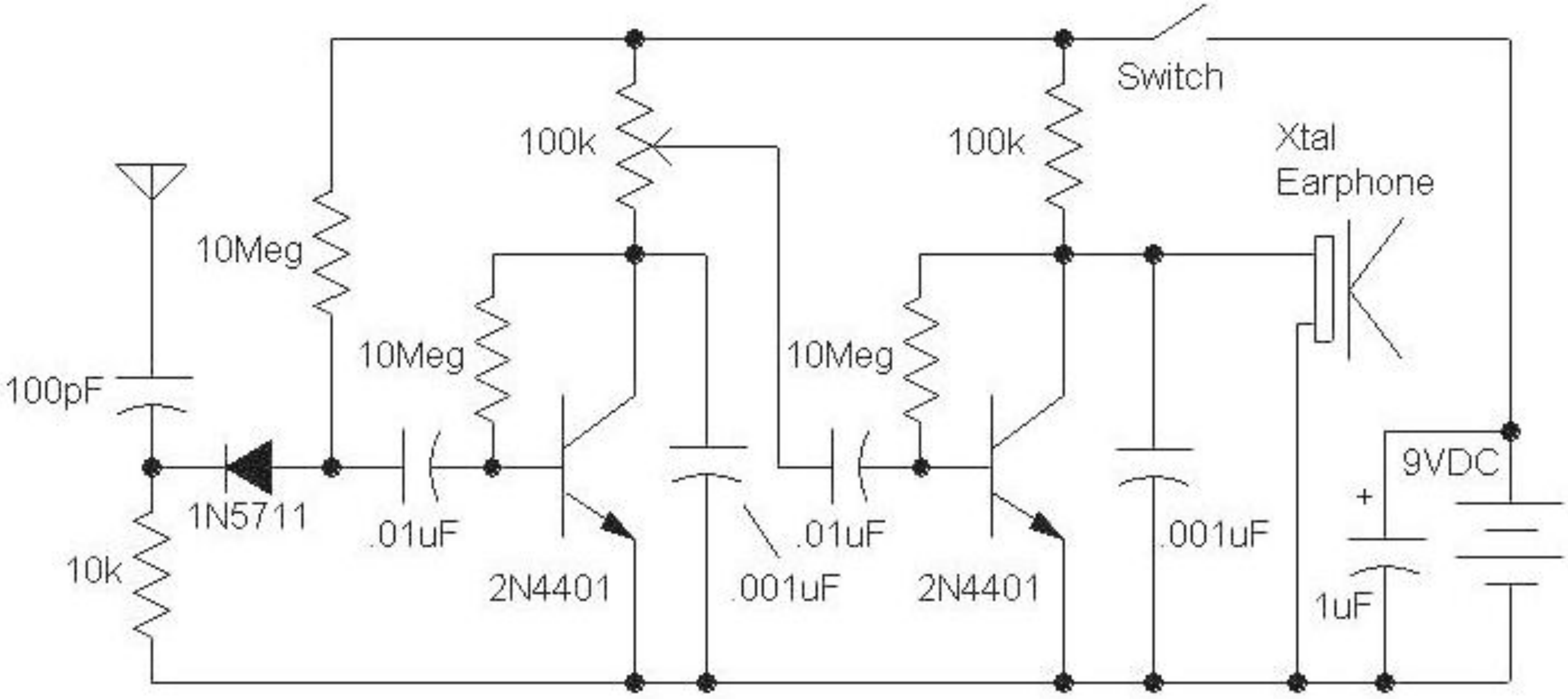


Multi-loop Circuits

- To analyze multi-loop circuits, we must apply both Kirchhoff's Loop Rule and Kirchhoff's Junction Rule
- To analyze a multi-loop circuit
  - Identify all complete loops and all junction points in the circuit
  - Apply Kirchhoff's Rules to these parts of the circuit separately
- Analyzing the single loops and the junctions will give us a set of coupled equations in several unknown variables
- We can solve these equations to get the quantities of interest using various methods
  - Direct substitution
  - Matrices and determinants

Kirchhoff’s Rules – A Radio Circuit

- More complicated circuits can be understood by studying all loops and all junctions
- The diagram below shows one of the simplest practically useful circuit, a tunable AM radio



A Complicated Circuit With IC Chip