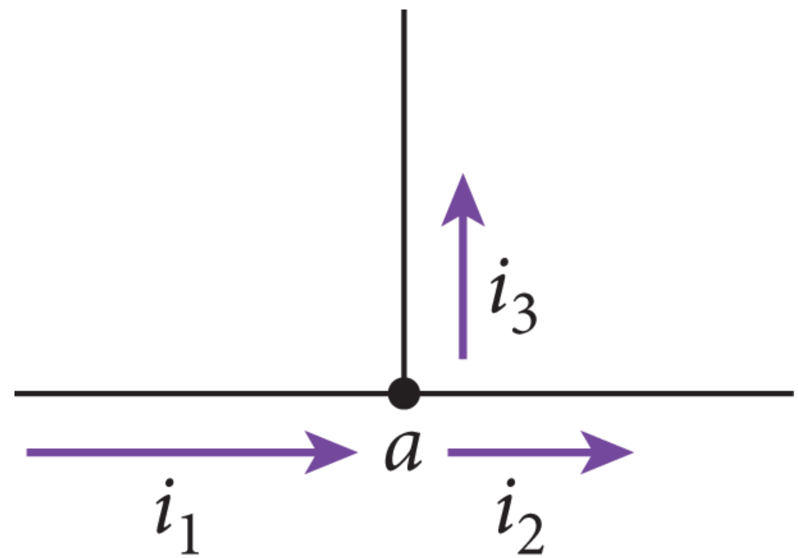
# Direct Current Circuits

# Kirchhoff's Junction Rule

- **The sum of the currents entering a junction must equal the sum of the currents leaving the junction.**

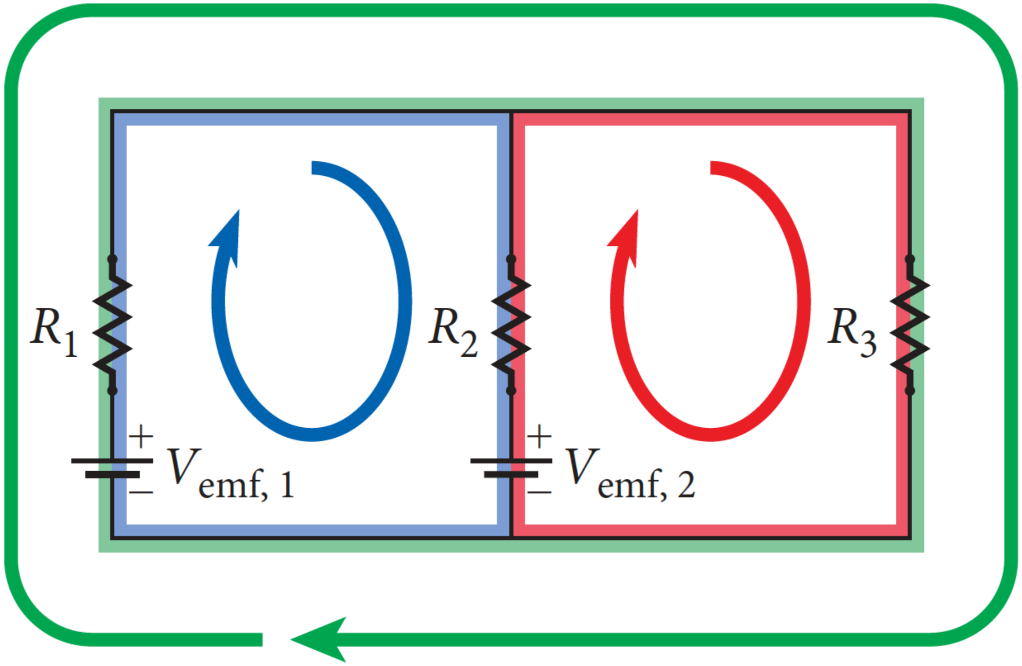
$$\text{Junction: } \sum_{k=1}^{n} i_k = 0$$

- A junction is a place in a circuit where three or more wires are connected to each other
- To start, assume direction for each currents:
  - Positive result: current flows in the assumed direction
  - Negative result: current flows in the opposite direction



# Kirchhoff's Loop Rule

- **The potential difference around a complete circuit loop must sum to zero.**
- A loop in a circuit is any set of connected wires and circuit elements forming a closed path.
- The sign for voltage sources and resistors depends on the analysis direction and the current direction



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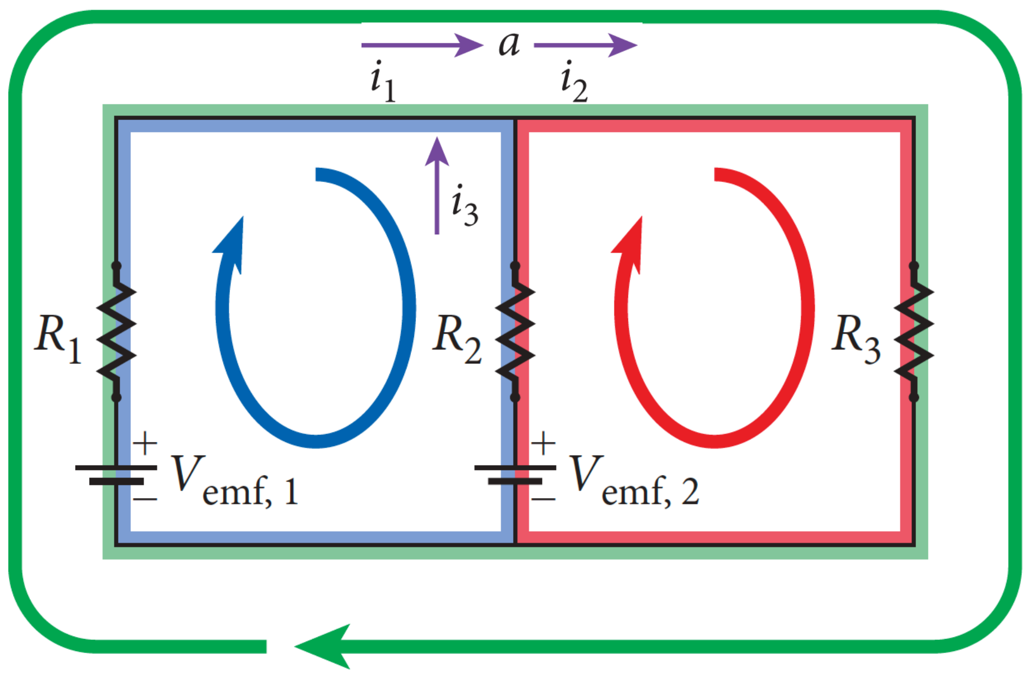
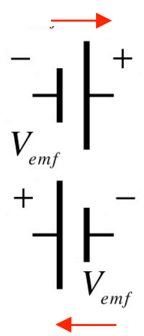
$$V_{emf,1} - i_1 R_1 + i_3 R_2 - V_{emf,2} = 0$$

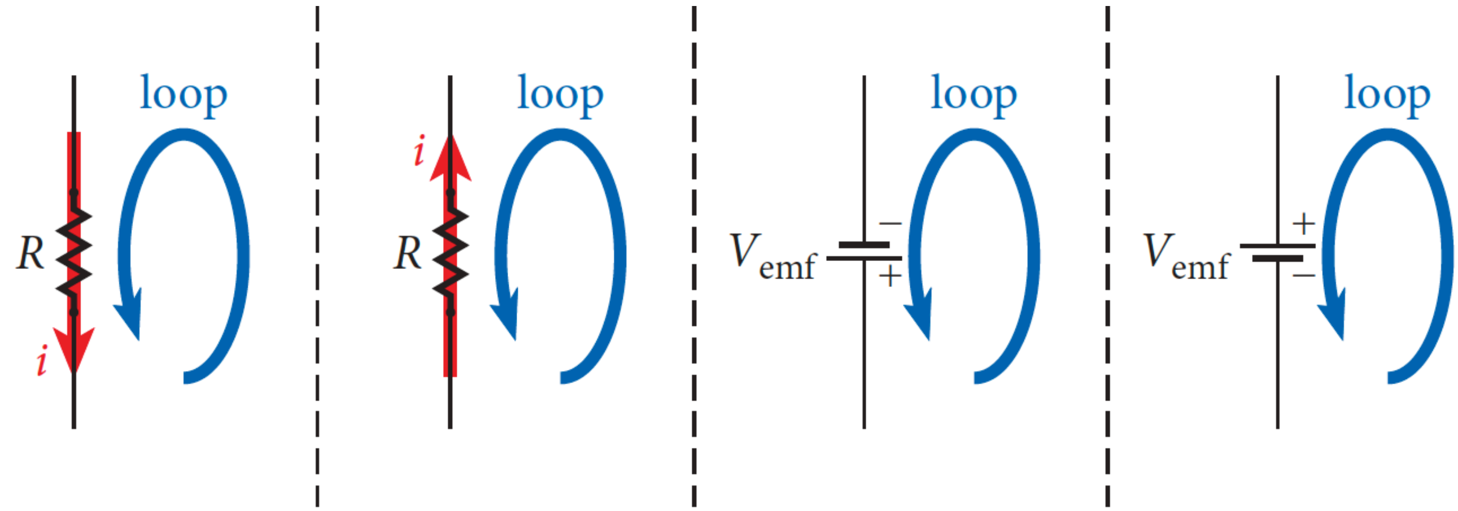
$$V_{emf,2} - i_3 R_2 - i_2 R_3 = 0$$



# Sign Convention for Potential Changes

| Element | Direction of Analysis | Potential Change | |
|---|---|---|---|
| $R$ | Same as current | $-iR$ | (a) |
| $R$ | Opposite to current | $+iR$ | (b) |
| $V_{emf}$ | Same as emf | $+V_{emf}$ | (c) |
| $V_{emf}$ | Opposite to emf | $-V_{emf}$ | (d) |




$\Delta V = -iR$ (a)

$\Delta V = +iR$ (b)

$\Delta V = +V_{emf}$ (c)

$\Delta V = -V_{emf}$ (d)

# Analysis of Single-loop Circuits

- Choose a direction for the current
- We can determine if our assumption for the direction of the current is correct after the analysis is complete
- Resulting current positive
  - Current is flowing in the same direction as we had chosen
- Resulting current negative
  - Current is flowing in the direction opposite to what we had chosen
- We can choose the direction in which we analyze the circuit
  - Any direction we choose will give us the same information

# Charging a Battery

- A 12.0 V battery with internal resistance $R_i = 0.200\ \Omega$ is being charged by a battery charger that is capable of delivering a current of magnitude $i = 6.00$ A

**PROBLEM**

- What is the minimum emf the battery charger must have to charge the battery?

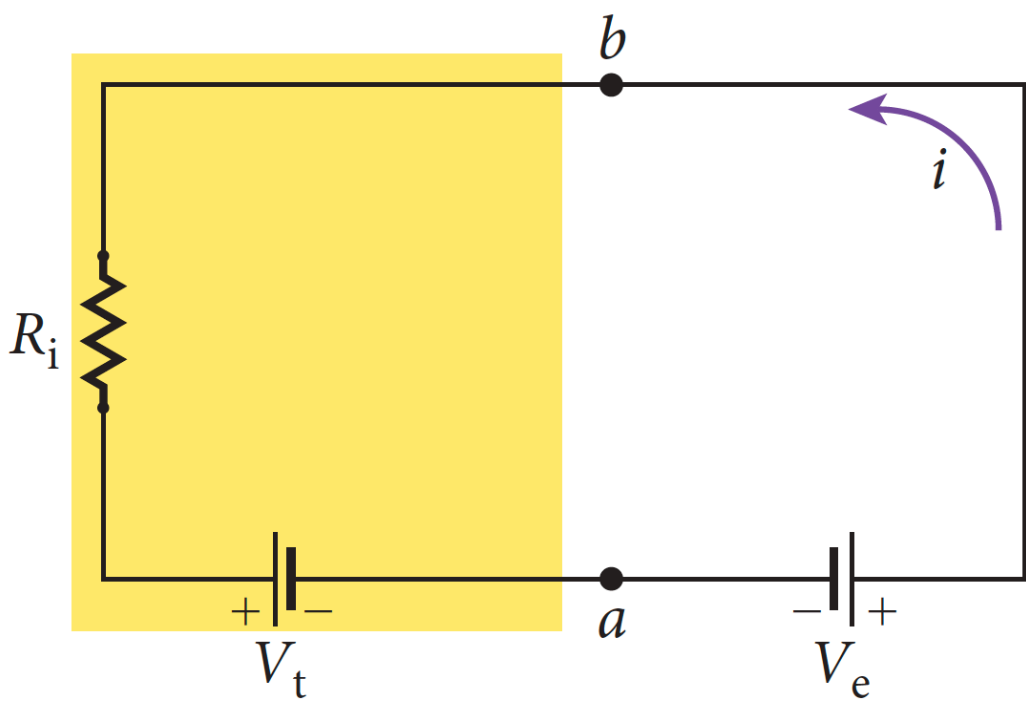
**SOLUTION**

**THINK**

- The battery charger must have enough potential difference to overcome the potential difference of the battery and the potential drop across the internal resistance of the battery
- The battery charge must be connected so that its positive terminal is connected to the positive terminal of the battery

# Charging a Battery

- We treat the internal resistance of the battery as a resistor in a single loop circuit that contains two sources of emf with opposite polarities
- **SKETCH**



# Charging a Battery

**RESEARCH**

- We apply Kirchhoff's Loop Rule
- We assume a current flowing in a counterclockwise direction
- The potential changes around the circuit must sum to zero
- Starting at point *b* we have

$$-iR_1 - V_t + V_e = 0$$

**SIMPLIFY**

- We solve this equation for the required potential difference of the charger

$$V_e = iR_1 + V_t$$

**CALCULATE**

- Putting in the numerical values

$$V_e = iR_1 + V_t = (6.00\text{ A})(0.200\ \Omega) + 12.0\text{ V} = 13.20\text{ V}$$

# Example – Battery Charger

- Two ideal batteries provide $V_1$=12V and $V_2$=6.0V and the resistors have $R_1$=4.0 Ω and $R_2$=8.0 Ω.

  (a) What is the current $i$?

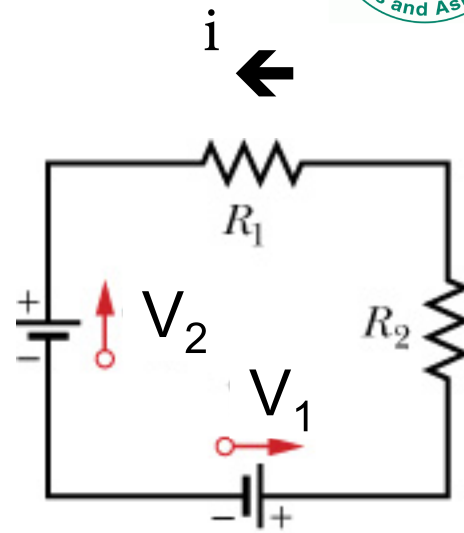
$$V_1 - iR_2 - iR_1 - V_2 = 0$$

$$i = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\Omega + 8.0\Omega} = 0.50\ \text{A}.$$

  (b) What is the power dissipated in $R_1$ and $R_2$?

$$P_1 = (0.5A)^2(4.0\Omega) = 1.0W$$

$$P_2 = (0.5A)^2(8.0\Omega) = 2.0W$$



# Example – Battery Charger

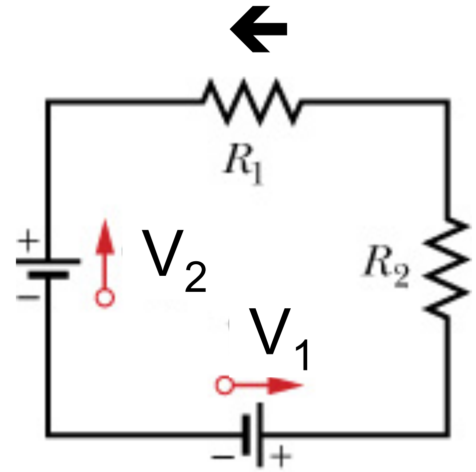
- Two ideal batteries provide $V_1$=12V and $V_2$=6.0V and the resistors have $R_1$=4.0 Ω and $R_2$=8.0 Ω.

(c) Is energy being supplied or absorbed by battery 1 and battery 2?

For battery 1, the emf arrow points in the same direction as the current: The battery supplies energy to the circuit.

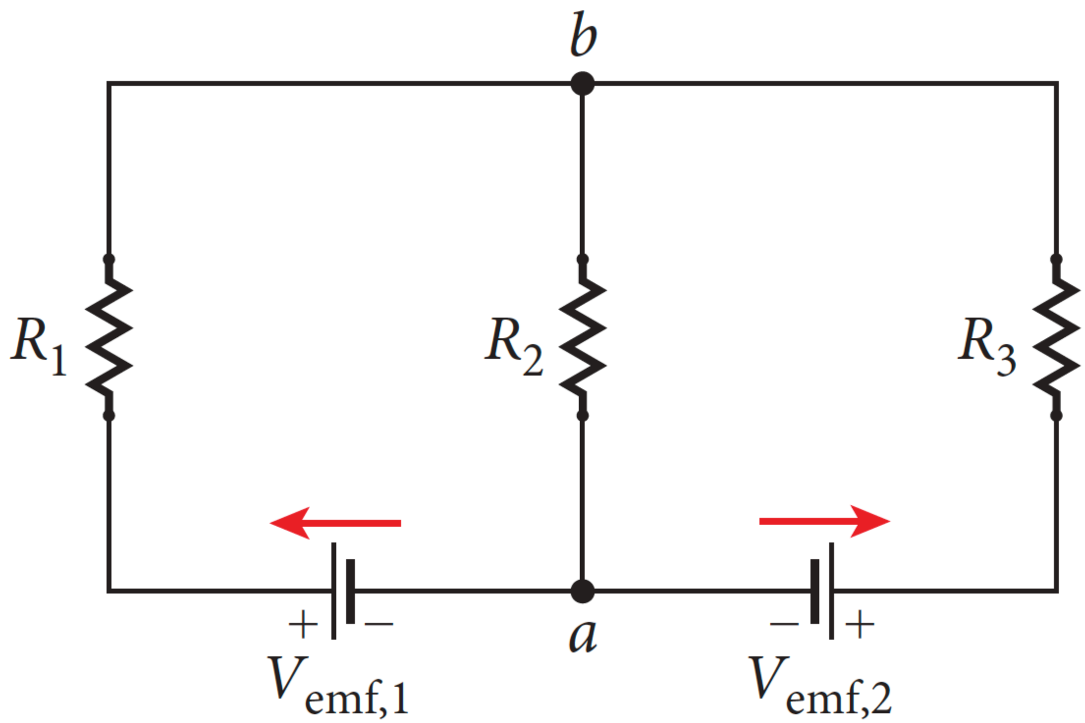
For battery 2, the emf arrow points opposite to the current direction: The battery absorbs energy from the circuit, it is being charged!

i = +0.5 A



# Multi-loop Circuit

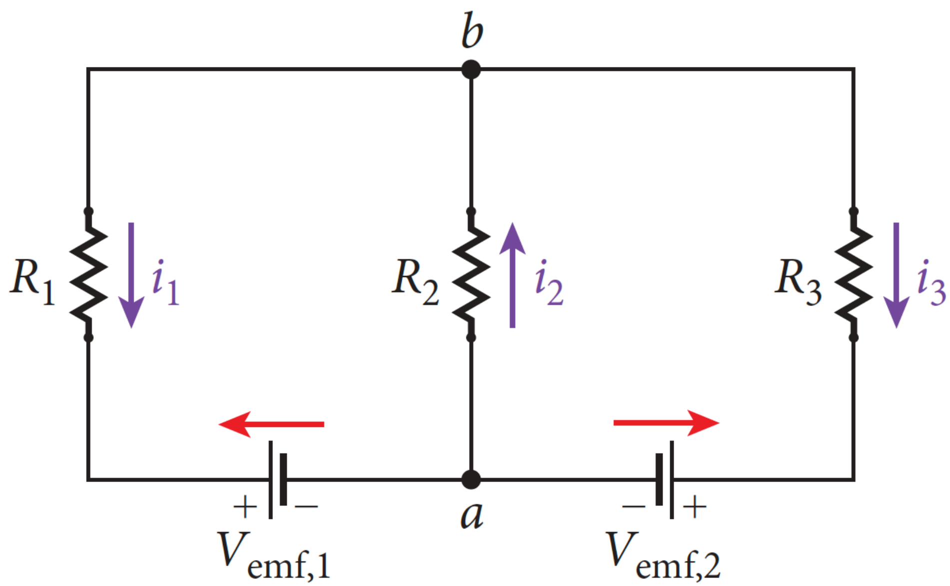
- Consider a circuit that has three resistors, $R_1$, $R_2$, and $R_3$ and two sources of emf, $V_{emf,1}$ and $V_{emf,2}$




- This circuit cannot be resolved into simple series or parallel structures

# Multi-loop Circuit

- To analyze this circuit, we need to assign currents flowing through the resistors
- We can choose the directions of these currents arbitrarily




- Junction *b* gives us

$$i_2 = i_1 + i_3$$

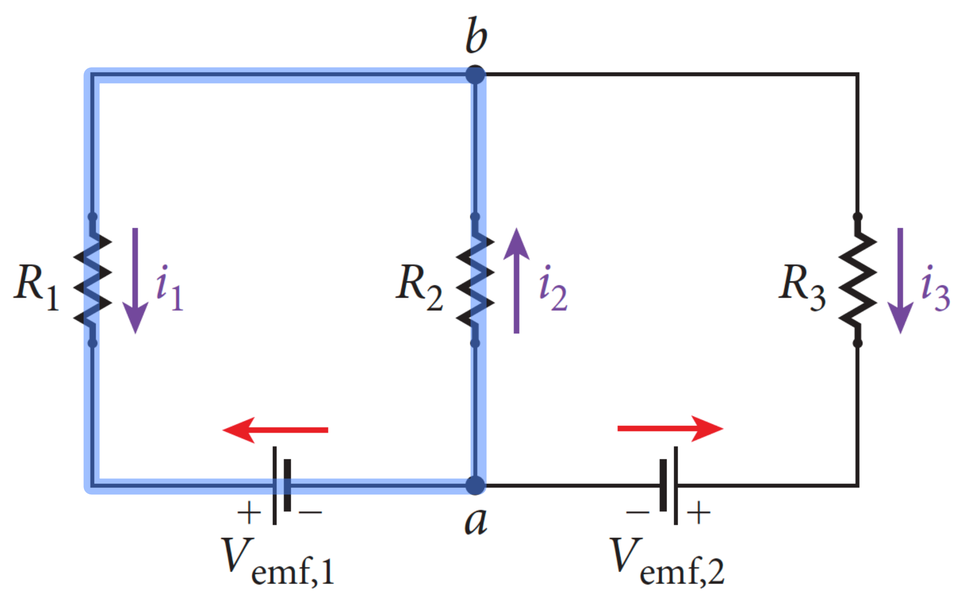
- Junction *a* gives us

$$i_1 + i_3 = i_2$$

- Which gives us no new information

# Multi-loop Circuit

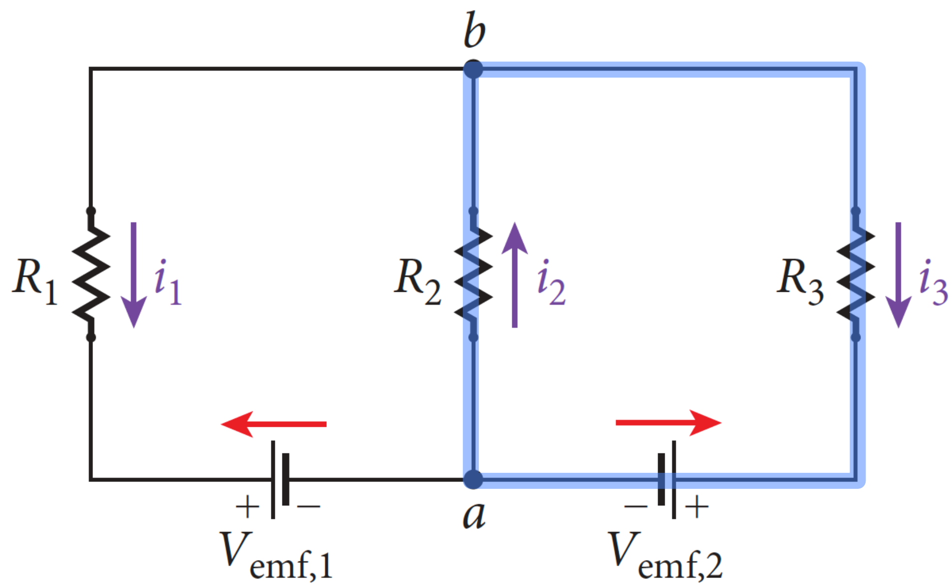
- At this point we have one equation and three unknowns from Kirchhoff's Junction Rule
- To get the necessary two additional equations we apply Kirchhoff's Loop Rule
- Start with left half of circuit analyzing the loop in a counterclockwise direction starting at *b*




$$-i_1R_1 - V_{\text{emf},1} - i_2R_2 = 0 \;\Rightarrow\; i_1R_1 + V_{\text{emf},1} + i_2R_2 = 0$$

# Multi-loop Circuit

- Now analyze the right half of the circuit in a clockwise direction starting at *b*




$$-i_3R_3 - V_{\text{emf},2} - i_2R_2 = 0 \;\;\Rightarrow\;\; i_3R_3 + V_{\text{emf},2} + i_2R_2 = 0$$

- We can analyze the outer loop also, but we will not gain any additional information

# Multi-loop Circuit

- Our three unknowns are $i_1$, $i_2$, and $i_3$ and our three equations are

$$\left.\begin{array}{l} i_2 = i_1 + i_3 \\ i_1 R_1 + V_{\text{emf},1} + i_2 R_2 = 0 \\ i_3 R_3 + V_{\text{emf},2} + i_2 R_2 = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} i_1 - i_2 + i_3 = 0 \\ i_1 R_1 + i_2 R_2 = -V_{\text{emf},1} \\ i_2 R_2 + i_3 R_3 = -V_{\text{emf},2} \end{array}\right. \qquad \begin{pmatrix} 1 & -1 & 1 \\ R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -V_{emf,1} \\ -V_{emf,2} \end{pmatrix}$$

- Use Cramer's Rule to get $i_1$

$$i_1 = \frac{\begin{vmatrix} 0 & -1 & 1 \\ -V_{\text{emf},1} & R_2 & 0 \\ -V_{\text{emf},2} & R_2 & R_3 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 1 \\ R_1 & R_2 & 0 \\ 0 & R_2 & R_3 \end{vmatrix}} = \frac{-R_2 V_{\text{emf},1} + R_2 V_{\text{emf},2} - R_3 V_{\text{emf},1}}{R_2 R_3 + R_1 R_2 + R_1 R_3} = -\frac{(R_2 + R_3) V_{\text{emf},1} - R_2 V_{\text{emf},2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

- Can do the same for $i_2$ and $i_3$ (in the book)

# Math Reminder: Cramer's Rule

$$\begin{cases} ax + by + cz &= j \\ dx + ey + fz &= k \\ gx + hy + iz &= l \end{cases}$$

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \text{ and } z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}.$$

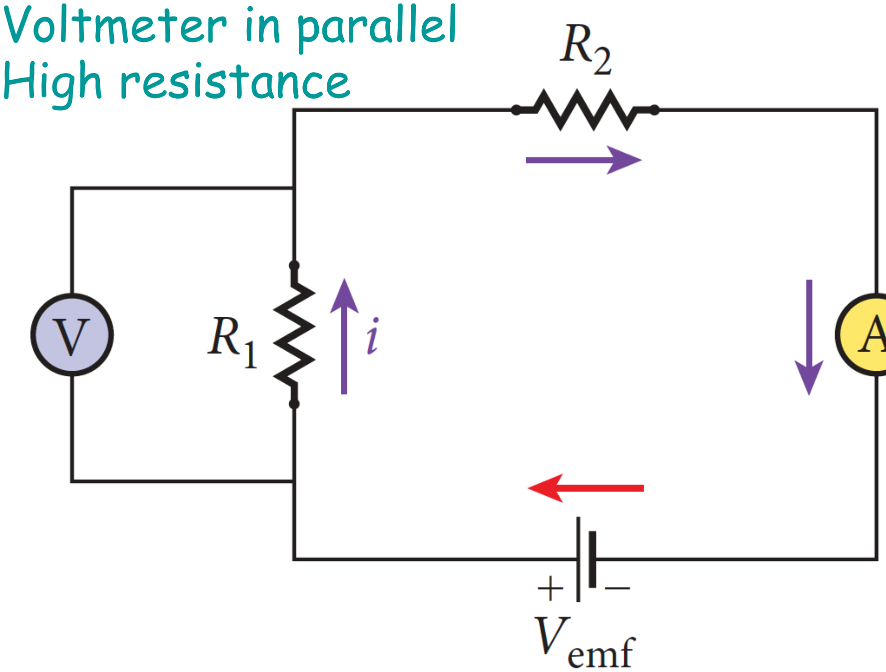
$$\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a(ei - hf) - d(bi - hc) + g(bf - ec)$$

# Ammeters and Voltmeters

- A device used to measure current is called an **ammeter**
- A device used to measure potential difference is called a **voltmeter**
- To measure the current, the ammeter must be placed in the circuit in *series*
- To measure the potential difference, the voltmeter must be wired in *parallel* with the component across which the potential difference is to be measured

Voltmeter in parallel
High resistance




Ammeter in series
Low resistance