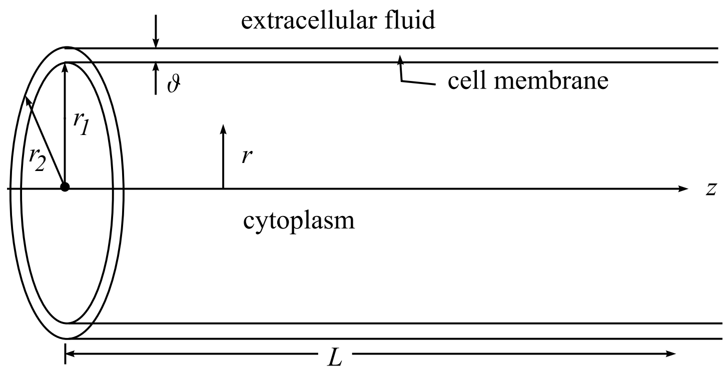
Resistance in the Nervous System

Nerve impulses are electrical currents in the form of ionic flows. It is therefore important to consider electrical resistance in the nervous system, and appropriate to do it here in connection with the discussions of Chapter 7. Capacitance in the nervous system was discussed on a Web page of Chapter 6, in the context of dielectric properties of the cell membrane.

The figure below is a schematic diagram of an isolated axon or dendrite of a nerve cell. The simplest model of this kind of nerve process is a cylinder filled with conducting fluid (the cytoplasm), enclosed by a thin annular cell membrane, with the whole surrounded by a conducting extracellular fluid. As shown in the figure, we take the z axis to be the cylinder axis. The transverse coordinate is r; it is measured perpendicular to the z axis.



The most important electrical properties of nerve cells are associated with propagation of transient signals (action potentials), which propagate in the axial direction of axons and dendrites. Action potentials are essentially time-dependent flows of Na^+ and K^+ ions through the cell membrane, i.e., in the transverse direction perpendicular to the cylinder axis. But nerve cells can also support steady-state flow of current in both the axial direction and the transverse direction. It is these steady-state currents we will discuss here.

A. Current flow in the axial direction through the cytoplasm of axons and dendrites

The ionic composition of cytoplasm is not far from that of sea water so we expect cytoplasmic resistivities to fall near the value $\rho(\text{sea water}) = 0.21\,\Omega\text{m}$, given in Table 7.1. We calculate two typical cases.

1. For a giant squid axon the average resistivity in the axial direction is $\rho_z(\text{squid axon}) = 0.5 \sim 1\Omega\text{m}$. Therefore for this kind of axon, whose radius is $r_1 = 0.5\,\text{mm}$, the axial resistance per unit length, $R_{\text{axial}}/L = \rho_z/\pi r_1^2$, is about $10^4\,\Omega/\text{cm}$.

2. For a mammalian neuron, $\rho_z(\text{mammalian axon}) = 2\,\Omega\text{m}$, so that for an axon whose radius is $r_1 = 5\,\mu\text{m}$, the axial resistance per unit length, $R_{\text{axial}}/L = \rho_z/\pi r_1^2$, is about $250\,\text{M}\Omega/\text{cm}$.

The resistivity values used in these calculations are averages for the axonal cytoplasm. In real cells, the cytoplasm is not just a homogeneous saline solution, but rather contains organelles (microtubules, mitochondria, etc.) and large molecules (proteins). Therefore, we can see why ρ_z for cytoplasm is close to but not the same as for sea water.

B. Current flow in the transverse direction through the cell membrane of axons and dendrites

Before discussing the transmembrane resistance we need the resistance, call it R_{tr}, for length L of a conducting cylindrical shell with inner and outer radii r_1 and r_2, respectively. We leave that as an interesting exercise. We will apply the result to the system shown in the figure, and call ρ_r the resistivity of the membrane for current flow in the transverse, or r, direction. The result, using the techniques of Example 2 in Chapter 7, is

$$R_{\mathrm{tr}} = \frac{\rho_r}{2\pi L} \ln\left(\frac{r_2}{r_1}\right) = \frac{\rho_r}{2\pi L} \ln\left(\frac{r_1 + \vartheta}{r_1}\right) \approx \frac{\rho_r \vartheta}{2\pi r_1 L}. \qquad (1)$$

In the last step we have made the approximation $\vartheta \ll r_1$, and expanded the logarithm according to $\ln(1 + \vartheta/r_1) \approx \vartheta/r_1$.

A notational comment is now in order. Neuroscientists usually call the numerator of the last relation in Eq. (1) the"specific membrane resistance", and write for it $R_m = \rho_r \vartheta$. Experimental values for R_m range from about 10^3 to $10^5\,\Omega\mathrm{cm}^2$. Therefore if we take for the membrane thickness $\vartheta \approx 3\,\mathrm{nm}$, the usual value, the resistivity for transmembrane ionic flow ranges from $\rho_r = 3 \times 10^9\,\Omega\mathrm{m}$ to $3 \times 10^7\,\Omega\mathrm{m}$. This is about the same as the resistivity of an insulator like wood (for which see Table 7.1).

If we now take Eq. (1) with $\vartheta \approx 3\,\mathrm{nm}$, and $L = 1\,\mathrm{cm}$, we obtain the following values for the transmembrane resistance (R_{tr}) of a section of axon 1 cm long:

1. Squid axons have a low transmembrane resistivity; for them $\rho_r = 2 \times 10^7\,\Omega\mathrm{m}$. Therefore, for a squid axon whose radius is $r_1 = 0.5\,\mathrm{mm}$, the result is $R_{\mathrm{tr}} = 1.8 \times 10^3\,\Omega$.

2. For a mammalian axon, we use $\rho_r = 3 \times 10^8\,\Omega\mathrm{m}$, which is the geometric mean of the range given in the previous paragraph. If the axon's radius is $r_1 = 5\,\mu\mathrm{m}$, the result is $R_{\mathrm{tr}} = 3 \times 10^6\,\Omega$.

If the values just calculated for the transmembrane resistances in these two systems are compared with the axial resistances, which were calculated in A.1 and A.2 above, we see that both axons are much more "leaky" for charge flow in the transmembrane direction than in the axial direction.

In discussing the transmembrane currents with this simple model we have made many approximations. In real plasma membranes the ions travel through ion selective channels which penetrate the membrane; there are Na^+ channels and K^+ channels, each with characteristic electrical and chemical properties. In the model we've used, these are averaged over. Real membranes are often sheathed in an insulating (myelin) layer so that ionic transport can only take place in small nodes between the myelin sheaths. More sophisticated models of nerve cell conduction take account of these and many other details but approximations are inevitable in treating real living systems.

Reference

M. J. Zigmond, et al., *Fundamental Neuroscience*, (Academic Press, San Diego, CA, 1999).