7. Electric Current
Chapter Summary

• Current I in a wire is defined by $I = dQ/dt$. Current density $\mathbf{J}(\mathbf{x})$ at a point $\mathbf{x}$ in a volume flow of charge is defined by $\mathbf{J} \cdot d\mathbf{A} = dI$; that is, $J_i(\mathbf{x})$ is the current per unit area in the ith direction at $\mathbf{x}$. Surface current density $\mathbf{K}(\mathbf{x})$ is the current per unit transverse length on a surface.

• If charges q with number density n move with mean velocity $\langle \mathbf{v} \rangle$, then the current density is $\mathbf{J} = qn\langle \mathbf{v} \rangle$.

• The continuity equation expresses local conservation of charge. In differential form,

$$\boldsymbol{\nabla} \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t};$$

in integral form,

$$\oint_S \mathbf{J} \cdot d\mathbf{A} = -\frac{dQ_{\text{enclosed}}}{dt}.$$

• ***Ohm's law.*** For current I in a wire with potential difference V,

$$V = IR$$

where R is the resistance. The more general, local version of Ohm's law is $\mathbf{J}(\mathbf{x}) = \sigma \mathbf{E}(\mathbf{x})$ where σ is the conductivity of the material. The resistance of a wire of length ℓ and cross section A is $R = \rho \ell / A$ where $\rho = 1/\sigma =$ resistivity.

• ***Joule's law.*** The power dissipated in a resistor is $P = IV = I^2 R$. The power density (with units of $\mathrm{W/m^3}$) dissipated in resistance by a current density $\mathbf{J}(\mathbf{x})$ in a material with conductivity σ is

$$\frac{dP}{dv} = \mathbf{J} \cdot \mathbf{E} = \sigma E^2 \quad \text{where} \quad dv = \text{volume.}$$