3. Basic Principles of Electrostatics

Self-test questions

1. Find the electric field \( \mathbf{E}(\mathbf{x}) \) and potential \( V(r) \) due to a uniformly charged sphere of radius \( a \) and total charge \( Q \).

2. A semi-infinite uniformly charged wire is located on the negative \( z \) axis. Find the electric field on the positive \( z \) axis.

3. What is the charge distribution if the electric field is \( \mathbf{E}(\mathbf{x}) = K \mathbf{x} / (r^2 + a^2)^{3/2} \)?

4. A pointlike electric dipole is located at the origin and points in the \( \mathbf{k} \) direction. The dipole moment is \( \mathbf{p} = p_0 \mathbf{k} \). Compare the electric field at the two points \((d, 0, 0)\) and \((0, 0, d)\).

5. Coulomb’s law states that the electrostatic force between two point charges is inversely proportional to \( r^2 \). Give two examples in which this law is tested for atomic dimensions or smaller.

6. A clever trick with superposition

A hole of radius \( R \) whose center is at the origin is cut from the \( xy \) plane. The rest of the \( xy \) plane has constant surface charge density \( \sigma \) (measured in coulombs/meter\(^2\)).

(a) What is \( \mathbf{E} \) on the \( z \) axis?

(b) Expand \( E_z(0, 0, z) \) for \( z \gg R \), and find the first two nonvanishing terms in the expansion.

For the next problem, computer graphics may help.

7. Four equal charges \( Q \) are located at the vertices of a square,

\[
(a, a), (-a, a), (-a, -a), (a, -a).
\]

Make a contour plot or a 3D surface plot of the potential in the \( xy \) plane. How much work is done in moving a test charge \( q \) from \((0, 0)\) to \((a, 0)\)? Is there an energy barrier?